# WINTER SCHOOL ON THEORY AND PRACTICE OF OPTIMAL STOPPING AND FREE BOUNDARY PROBLEMS

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# WORKSHOP ON STOCHASTIC ANALYSIS, CONTROL AND MATHEMATICAL FINANCE

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Book of Abstracts (alphabetical order)

**Speaker:** Andrew Allan (ETH Zurich)
**Title:** Parameter Uncertainty in Stochastic Filtering
**Abstract:** We consider the problem of filtering - that is, estimating the current state of a stochastic 'signal' process from noisy observations - under uncertainty of both the dynamics of the signal and of its relationship with our observations. We take a nonlinear expectations approach, which leads naturally to a pathwise stochastic optimal control problem, the solution of which provides a new way of 'learning' unknown parameter values dynamically through time.

**Speaker:** Cheng Cai (University of Leeds)
**Title:** Optimal hedging for American put options with a single trade
**Abstract:** In the Black-Scholes model, an option seller constructs a self-financing stock-bond portfolio in order to Delta hedge a short position in a perpetual American put option. In contrast to the continuous trading of the Black-Scholes model, the option seller can only rebalance her portfolio once before the time $\tau_a \wedge \tau_b$ at which the underlying stock price $S_t$ leaves an interval $(a, b)$. Here the lower end-point $a$ is the optimal exercise price of the put option and $b(> a)$ is chosen arbitrarily. The goal is to determine the optimal time to rebalance the portfolio and the optimal hedge ratio (stock holding after the trade) that minimize the variance of the so-called tracking error (at time $\tau_a \wedge \tau_b$). First, we formulate the optimal hedging problem for a fixed initial stock holding as a one dimensional optimal stopping problem. This is solved by constructing three different free boundary problems depending on possible parameter choices. Second, we study analytically how the stopping boundaries move in response to variations in the initial stock holding. Finally, we obtain an equation that must be satisfied by an optimal initial stock holding.

**Speaker:** Konstantinos Dareiotis (University of Leeds)
**Title:** Approximation of stochastic equations with irregular drift
**Abstract:** One of the remarkable results in stochastic analysis is that the presence of noise in certain dynamical systems has a regularizing effect. The simplest example is the following. For $f \in C^\alpha$, the differential equation $\dot{x} = f(x)$ is in general not well posed when $\alpha \in (0, 1)$ (might have infinitely many solutions). However, adding white noise to the equation leads to well posedness, that is, the equation $\dot{x} = f(x) + \dot{w}$ has a unique solution for every initial condition. Recently, the numerical approximation of stochastic equations with such irregular drift has attracted the attention of many researchers. In a series of articles it was shown that if $f \in C^\alpha$, for $\alpha \in (0, 1)$, then the strong rate of convergence of the Euler-Maruyama is $\alpha/2$. In this talk we will show that if one exploits the noise in an efficient way it can be shown that the rate of convergence is in fact much better than $\alpha/2$. Namely, we will see that the rate is $1/2$ provided that $f$ is only

Dini continuous, while in dimension $d = 1$, the same conclusion holds for $f$ merely measurable and bounded. In addition, we will see that if $f \in C^\alpha$, then the rate of convergence is in fact $(1+\alpha)/2$. Our novel approach can also be applied to non-Markovian settings such as equations driven by fractional Brownian motion where PDE techniques are not available.

This is a joint work with Oleg Butkovsky and Máté Gerencsér.

**Speaker:** Erik Ekström (University of Uppsala)
**Title:** A Bayesian fraud detection problem
**Abstract:** We set up and analyse a simple Bayesian model of fraud detection. The problem is formulated as an asymmetric stochastic game between a fraudster who controls the rate at which information is stolen and a stopper who chooses a time to clean the system.

**Speaker:** Abel Guada Azze
**Title:** Optimal exercise for American options under pinning effect
**Abstract:** We address the problem of optimally exercising American options based on the assumption that the underlying stock's price follows a Brownian bridge whose final value coincides with the strike price. In order to do so we solve, for put options, a discounted optimal stopping problem via the free-boundary problem approach, and then we show how to easily obtain the solution for the call option case by establishing a put-call parity. This work comes up as a first approach of optimally exercising an option within the so-called "stock pinning effect". The stopping boundary for this problem is proved to be the unique solution, up to certain regularity conditions, of an integral equation, which is numerically solved by a recursive fixed point algorithm. When the process' volatility is unspecified, we provide an estimated optimal stopping boundary that, alongside pointwise confidence intervals, suggest alternative stopping rules. Our method is compared, using real data exhibiting the so-called stock pinning effect, with the optimal exercise time based on a geometric Brownian motion.

**Speaker:** Sonya Javadi (Dogus University)
**Title:** An Optimal Stopping Approach for the End-of-Life Inventory Problem
**Abstract:** In this talk, the end-of-life inventory problem for the supplier of a product in its final phase of the service life cycle is introduced. In the literature, final phase is a part of product life cycle in which starts when the production of items terminates and continues until the last sold item warranty is expired. The supplier places a final order for spare parts at the beginning of this phase to serve the arriving customers. Over the final phase, the supplier may completely change the strategy and switch to an alternative policy which is remarkably cheaper than continuing the current

situation. The alternative policy may be defined as replacing the defective items with substitutable products or offering the customer a discounted new generation one. The supplier as a decision-maker faces a bi-level optimization problem. In this problem, the final order quantity and switching time to an alternative policy are decision variables which will minimize the total expected discounted cost function. The switching time is a stopping time according to a realization of the arrival process of defective items. To that end, it is assumed the customers are arriving according to a non-homogenous Poisson process. Furthermore, in this talk how this optimal stopping problem is solved will be discussed in the detail. At the end some numerical examples are given to make the problem and solving method more clear.

**Speaker:** Junkee Jeon (Kyung Hee University)
**Title:** Optimal Insurance with Limited Commitment in a Finite Horizon
**Abstract:** We study a finite horizon optimal contracting problem of a risk-neutral principal and a risk-averse agent who receives a stochastic income stream when the agent is unable to make commitments. The problem involves an infinite number of constraints at each time and each state of the world. Miao and Zhang (2015) have developed a dual approach to the problem by considering a Lagrangian and derived a Hamilton-Jacobi-Bellman equation in an infinite horizon. We consider a similar Lagrangian in a finite horizon, but transform the dual problem into an infinite series of optimal stopping problems. For each optimal stopping problem we provide an analytic solution by providing an integral equation representation for the free boundary. We provide a verification theorem that the value function of the original principal's problem is the Legender-Fenchel transform of the integral of the value functions of the optimal stopping problems. We also provide some numerical simulation results of optimal contracting strategies

**Speaker:** Damien Lamberton (Université Paris Est - Marne-la-Vallée)
**Title:** Variational inequalities in the Heston model
**Abstract:** In this talk, based on joint work with Giulia Terenzi, I will discuss the use of variational inequalities to treat American option prices in the Heston model. The analysis of the associated variational inequality will be based on the approach used in recent papers by Daskalopoulos and Feehan (2011) and Feehan and Pop (2015). In particular, we will show how the affine structure of the model simplifies the identification of the semigroup associated with a bilinear form as the transition semigroup of the underlying diffusion.

**Speaker:** Younhee Lee (Chungnam National University)
**Title:** Real option under a regime-switching jump-diffusion model on finite time horizon

**Abstract:** We consider a regime-switching jump-diffusion model to deal with a real option on finite time horizon. In this talk, we focus on determining an optimal investment time to maximize the discounted expectation of a payoff function when a cash flow is given by the regime-switching jump-diffusion model. Numerical experiments are carried out to show a number of phenomena of the real option involving jumps.

**Speaker:** Marcos Leutscher (ENSAE ParisTech)
**Title:** Mean-Field Games with optimal stopping and continuous control: a relaxed solution approach
**Abstract:** We consider the mean-field game where each agent determines the optimal stopping time and continuous control to maximize a functional depending on the density of the state processes of agents still present in the game and their control. We suppose also that the density of players can influence the dynamics of each player. We place ourselves in the framework of relaxed optimal stopping developed by Bouveret, Dumitrescu and Tankov, which amounts to looking for the optimal occupation measure of the stopper and his control rather than the optimal stopping time and continuous control. This framework allows us to prove the existence of the relaxed Nash equilibrium and the uniqueness of the associated value of the representative agent under mild assumptions. Finally, we prove a rigorous relation between relaxed Nash equilibria, the Nash equilibria in the usual sense and the notion of mixed solutions introduced in earlier works on the subject.

**Speaker:** Ruiqi Liu (University of Warwick)
**Title:** A Mathematical Model to Technical Analysis
**Abstract:** Among various methods in the area of technical analysis, we study a model capturing the support and resistance reversal feature for a stock price process. We solve the optimal stopping problem with the payoff to be a discounted utility function, which gives the optimal trading strategies to the selling problem. We further present the solution to the corresponding purchase problem.

**Speaker:** Nikita Merkulov (University of Leeds)
**Title:** Value of an asymmetric information optimal stopping game
**Abstract:** We study an optimal stopping game of two players. A scenario - a "state of the world" in which the game is played - is chosen according to a probability distribution known to both players. Immediately after the game starts, one of the players receives the information about the scenario, therefore knowing exactly the structure of the game.

We restrict ourselves to a special case when only the terminal time payoff depends on the random scenario. We introduce an equivalent game of optimal controls, and further consider a game in which one of the players is only

allowed to use absolutely continuous controls. Under certain assumptions on continuity and order of the payoff processes, we show that the latter game has a value in randomized strategies. We then construct an appropriate approximation of an arbitrary control by a sequence of Lipshitz-continuous controls, and deduce that the initial game has a value in randomized strategies.

**Speaker:** Alessandro Milazzo (Imperial College London)
**Title:** Optimal stopping for the exponential of a Brownian bridge
**Abstract:** In this paper we study the problem of stopping a Brownian bridge $X$ in order to maximise the expected value of an exponential gain function. The problem was posed by Ernst and Shepp in their paper [*Commun. Stoch. Anal.*, **9** (3), 2015, pp. 419-423] and was motivated by bond selling with non-negative prices. Due to the non-linear structure of the exponential gain, we cannot rely on methods used in the literature to find closed-form solutions to other problems involving the Brownian bridge. Instead, we must deal directly with a stopping problem for a time-inhomogeneous diffusion. We develop techniques based on pathwise properties of the Brownian bridge and martingale methods of optimal stopping theory, which allow us to find the optimal stopping rule and to show regularity of the value function.

**Speaker:** José Manuel Pedraza Ramirez (London School of Economics)
**Title:** Predicting in a $L^p$ sense the last zero of a spectrally negative Lévy process.
**Abstract:** Given a spectrally negative Lévy process drifting to infinity, we are interested in the last time $g$ in which the process is below zero. At any time $t$, the value of $g$ is unknown and it is only with the realisation of the whole process when we can know when the last zero of the process occurred. However, this is often too late, we usually are interested in know how close is the process to $g$ at time $t$ and take some actions based on this information. We are interested on finding a stopping time which is as close as possible to $g$ (in a $L^p$ distance). We prove that solving this optimal prediction problem is equivalent to solve an optimal stopping problem in terms of a two dimensional Markov process which involves the time of the current excursion away from the negative half line and the Lévy process. We state some basic properties of the last zero process and prove the existence of the solution of the optimal stopping problem. Then we show the solution of the optimal stopping problem (and therefore the optimal prediction problem) is given as the first time that the process crosses above a non-increasing and non-negative curve dependent on the time of the last excursion away from the negative half line.

**Speaker:** Goran Peskir (University of Manchester)
**Title:** Optimal real-time detection of a drifting Brownian coordinate
**Abstract:** TBA

**Speaker:** Benjamin Robinson (University of Bath)
**Title:** An SDE with no strong solution arising from the stochastic control of martingales in a radially symmetric environment
**Abstract:** We consider a control problem for multidimensional martingales in a radially symmetric environment. We find that there are only two possibilities for the optimal behaviour of the controlled process: either the process moves radially, acting as a Brownian motion on the radius of the domain, or it moves on a tangent to its current position, pushing out away from the origin. The control problem thus reduces to a one-dimensional switching problem, which we solve using the theory of viscosity solutions for HJB equations. In this talk, we focus on a class of cost functions for which we can show that tangential behaviour is a strict minimiser of the cost at the origin. This leads us to investigate SDEs that describe this tangential behaviour. We show that, under additional conditions, such an SDE has no strong solution started from the origin. This SDE has similar properties to Tsirelson's example of an SDE with no strong solution, and our proof builds on Emery and Schachermayer's work that relates Tsirelson's SDE to a Brownian motion on a circle. Our results on an SDE with no strong solution suggest that the value of the control problem in the strong sense differs from the value in a suitable weak sense.

This is joint work with Alexander Cox.

**Speaker:** Bhudisaksang Theerawat (University of Oxford)
**Title:** Online drift estimation of Lévy driven diffusion
**Abstract:** Our main objective is to show the convergence of online drift estimation of a jump diffusion driven by Lévy process. For a diffusion driven by only Brownian motion, the proof of the convergence of online parameter estimation relies on the existence of a classical solution of a corresponding Poisson equation, which is required for the use of Itô lemma to control the deviation from the descent direction. However, for the jump case, there is no existing result regarding a classical solution of a non-local Poisson equation. We propose a novel method to overcome this difficulty. We bypass the use of a classical Itô lemma by showing that the non-local Poisson equation has a solution in a martingale sense. Then, we apply an extended Itô formula to obtain that the deviation term can be written as a sum of a Riemann integral, a stochastic integral and a covariation term, which is used to show the convergence of the deviation term. This ultimately implies the convergence of the online drift estimation for a jump diffusion.

**Speaker:** Yuqiong Wang (University of Uppsala)
**Title:** Sequential Testing and Quickest Detection Problems for a Multi-Dimensional Wiener Process
**Abstract:** We study extensions to higher dimensions of the classical Bayesian sequential testing and detection problems for Brownian motion, with a special focus on general properties of the corresponding cost functions. In the main result we show that, for a large class of problem formulations, the cost function is unilaterally concave. This concavity result is then used to deduce structural properties for the continuation and stopping regions. We further discuss some different formulations and their specific properties.

This is joint work with Erik Ekström.

**Speaker:** Jingsi Xu (Univ. of Manchester)
**Title:** Optimal Mean-Variance Portfolio Selection with No Short-Selling Constrai
**Abstract:** In this paper, the objective is to study the continuous mean-variance portfolio selection with no short selling constraint and obtain a time-consistent solution. We assume that there is a self-financing portfolio with wealth process $X_t^u$, in which $u \geq 0$ represents the fraction of wealth invested in the risk asset under the short selling prohibition. We investigate the following mean-variance optimal constrained problem:

$$\sup_{u\geq 0}[\mathsf{E}_{t,x}(X_T^u) - c\mathsf{Var}_{t,x}(X_T^u)]$$

in which $c > 0$ is a constant and $t$ runs from 0 to the maturity time $T$. To envisage the quadratic non-linearity introduced by the variance, the method of Lagrangian multipliers reduces the nonlinear problem into a set of linear problems which can be solved by applying the Hamilton-Jacobi-Bellman equation and change of variables formula with local time on curves. Solving the HJB system gives the time-inconsistent solution and from there, we derive the time-consistent optimal control, which is given by:

$$u_*^d(t,x) = \max[\frac{\delta}{2c\sigma}\frac{1}{x}e^{(\delta^2-r)(T-t)}, 0]$$

in which $\delta = \frac{\mu-r}{\delta}$ is the related risk coefficient. In addition, two constrained problems obtained by imposing the lower/upper boundary on the expectation/variance of the terminal wealth are also solved.

**Speaker:** Mihalis Zervos (London School of Economics)
**Title:** A principal-agent model with costly renegotiation
**Abstract:** We consider a continuous-time principal-agent model without precommitment. The agent runs an economic project on behalf of the principal. To this end, the agent applies effort that is costly to them and unobservable by the principal. In return, the agent receives compensation from

the principal. The agent is strictly risk-averse and their objective is to maximise their expected utility of compensation minus their expected disutility of effort. The principal is risk-neutral and their objective is to maximise their expected utility of income generated by the project minus the compensation paid to the agent. The optimal contract should maximise the principal's expected utility subject to the constraint that it should induce a contractual environment in which it is optimal for the agent to always be truthful. To exclude the requirement of precommitment, the contract allows for costly renegotiation. The optimal contract is fully determined by deriving the explicit solution to a suitable stochastic control problem combining regular control of the drift and the volatility with singular control and optimal stopping.

**Speaker:** Jingjie Zhang (University of Michigan)
**Title:** On the Notions of Equilibria for Time-Inconsistent Stopping Problems in Continuous Time
**Abstract:** A new notion of equilibrium, which we call strong equilibrium, is introduced for time-inconsistent stopping problems in continuous time. Compared to the existing notions introduced in Time-Consistent Stopping Under Decreasing Impatience and On Finding Equilibrium Stopping Times for Time-Inconsistent Markovian Problems, which in this paper are called mild equilibrium and weak equilibrium respectively, a strong equilibrium captures the idea of subgame perfect Nash equilibrium more accurately. When the state process is a continuous-time Markov chain and the discount function is log sub-additive, we show that an optimal mild equilibrium is always a strong equilibrium. Moreover, we provide a new iteration method that can directly construct an optimal mild equilibrium and thus also prove its existence.

**Speaker:** Yufei Zhang (Oxford)
**Title:** Penalty schemes and policy iteration for stochastic hybrid control problems
**Abstract:** In this talk, we propose a penalty method for solving mixed optimal stopping and control problems. The solution and free boundary of an associated HJB variational inequality are constructed from a sequence of penalized equations, for which the penalization error is estimated. The penalized equation is then discretized by monotone approximations, and solved by an efficient iterative algorithm with superlinear convergence. We further extend the penalty scheme and its convergence analysis to stochastic hybrid control problems involving continuous and impulse controls. Numerical examples for infinite-horizon optimal switching problems are presented to illustrate the effectiveness of the penalty schemes.

This is joint work with Christoph Reisinger (Mathematical Institute, University of Oxford, United Kingdom).

**References.**
[1] C. Reisinger and Y. Zhang, *A penalty scheme and policy iteration for nonlocal HJB variational inequalities with monotone drivers*, arxiv:1805.06255, 2018.
[2] C. Reisinger and Y. Zhang, *Error estimates of penalty schemes for quasi-variational inequalities arising from impulse control problems*, SIAM J. Control Optim., forthcoming (arxiv:1901.07841, 2019).

**Speaker:** Christina Zou (University of Oxford)
**Title:** A free boundary representation of Root's and Rost's solutions to the Skorokhod embedding problem for Markov processes
**Abstract:** A classical problem in stochastic analysis is the Skorokhod embedding problem: Given a Brownian motion and a probability measure, the task is to stop the trajectories of the process such that the terminal points are distributed according to the given measure. One approach in order to determine a solution for the problem is to construct it as a first hitting time of a so-called barrier. Despite the vast amount of literature, only a few deals with the construction of such a barrier and most literature only consider one-dimensional diffusions. I am going to show a general free boundary characterisation of the classic solutions made by Root and Rost for a large class of right continuous standard Markov processes, which includes e.g. n-dimensional Brownian motion (or more general hypoelliptic diffusions) and jump diffusions.