UNIVALENT FOUNDATIONS MINI-COURSE: OUTLINE AND REFERENCES

SUMMER SCHOOL ON HIGHER TOPOS THEORY AND UNIVALENT FOUNDATIONS, LEEDS, JUNE 2019, https://conferences.leeds.ac.uk/httuf/

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1. Planned outline

The ordering/balancing of topics is very liable to change, but the basic selection of topics will remain roughly as given here.

Lecture 1: *Basics of homotopical foundations*
Basic infrastructure of working in HoTT/UF: univalence, HIT's. First steps in synthetic homotopy theory.

Lecture 2: *Truncation and connectivity.*
Definition of truncation (h-levels), connectivity; infrastructure for these. Freudenthal suspension theorem.

Lecture 3: *Modalities and Blakers–Massey.*
General modalities; definition, examples, theory. The Anel–Bidermann–Finster–Joyal generalised Blakers–Massey theorem.

Lecture 4: *Ordinary mathematics in the univalent world.*
Practical comparison of HoTT/UF with other popular foundations. "All traditional mathematics can be routinely translated into type theory." Constructivity issues.

Lecture 5: *Category theory in the univalent world.*
Category theory (possibly *higher*-) in the univalent setting: where "ordinary mathematics" meets "synthetic homotopy theory".

2. References

2.1. **Background.** Some familiarity with (pre-homotopical) dependent type theory, as introduced in any of the following texts, will be very helpful:

1. Nordström–Petersson–Smith, *Martin-Löf's type theory*, 2000, http://www.cse.chalmers.se/~bengt/papers/hlcs.pdf
2. Per Martin-Löf, *Intuitionistic Type Theory*, Bibliopolis, 1984, https://archive-pml.github.io/martin-lof/pdfs/Bibliopolis-Book-retypeset-1984.pdf
3. Ch.1 of *The HoTT book*, 2013, https://homotopytypetheory.org/book/

I'll quickly go over this material at the start to fix conventions and so on, but will assume the basic language is familiar.

Familiarity with basic ideas of homotopy theory will be very helpful, up to say homotopy groups and homotopy equivalences. I won't make formal use of these, but will refer to them for intuition when introducing their type-theoretic counterparts.

Date: June 8, 2019.

2.2. **Further reading on course topics.** The material of the course is essentially extracted from the following sources, which therefore provide good further reading:

- Mike Shulman and Dan Licata, *Calculating the fundamental group of the circle in homotopy type theory*, 2013, arxiv:1301.3443
- Ch.1–9 of *The HoTT book*, 2013, https://homotopytypetheory.org/book/
- Guillaume Brunerie, *On the homotopy groups of spheres in homotopy type theory* (PhD thesis), 2016, arxiv:1606.05916
- Favonia, Eric Finster, Dan Licata, Peter LeFanu Lumsdaine, *A mechanization of the Blakers–Massey connectivity theorem in homotopy type theory*, 2016, arxiv:1605.03227
- Mathieu Anel, Georg Biedermann, Eric Finster, André Joyal, *A generalized Blakers–Massey theorem*, 2017, arxiv:1703.09050
- Egbert Rijke, Mike Shulman, Bas Spitters, *Modalities in homotopy type theory*, 2017, arxiv:1706.07526
- Benedikt Ahrens, Chris Kapulkin, Mike Shulman, *Univalent categories and the Rezk completion*, 2013, arxiv:1303.0584
- Benedikt Ahrens, Peter LeFanu Lumsdaine, Vladimir Voevodsky, *Categorical structures for type theory in univalent foundations*, 2017, arxiv:1705.04310

2.3. **Models of type theory.** An important topic that I'll probably only say a little about in the course is *models* of type theory: that is, how constructions and theorems proved abstractly about types can be interpreted to give results about other mathematical structures such as sets, groupoids, or spaces.

The following is a few introductory suggestions, not at all comprehensive!

- Martin Hofmann, *Syntax and semantics of dependent types*, 1995: http://dx.doi.org/10.1017/CBO9780511526 Excellent introduction to models of type theory in general; and some specific "pre-homotopical" models.
- Chris Kapulkin, Peter LeFanu Lumsdaine, *The simplicial model of univalent foundations (after Voevodsky)*, 2012, arxiv:1211.2851. The first full homotopical model of type theory
- Chris Kapulkin, Peter LeFanu Lumsdaine, *The homotopy theory of type theories*, 2016, arxiv:1610.00037. See particularly the introduction, for a big-picture discussion of the relationships (proven and conjectured) between type theories and infinity-categories.
- Mike Shulman, *All* $(\infty, 1)$*-toposes have strict univalent universes*, 2019, arxiv:1904.07004, and preceding papers of Shulman, for the strongest current results on interpreting type theory in infinity-toposes.

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