Aggregate production functions: How does the Solow Residual change when introducing quality-adjusted values for capital, labour, and energy?

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Abstract

We review and discuss the use of quality-adjusted factors of production as inputs to macroeconomic production functions to model economic growth. First, we test the proposition that energy-augmented production functions are better able to explain economic output than traditional specifications that include unadjusted capital and labour only, by comparing the Solow Residual over the period 1960-2010 for the UK and Portugal. Second, we study the effect on the Solow Residual of introducing quality-adjusted variables for capital, labour, and energy, namely capital services, human capital, and useful work, respectively.

We model the Cobb-Douglas (CD) and Constant Elasticity of Substitution (CES) production functions; determining goodness-of-fit to historical economic growth, total factor productivity (TFP), the Solow Residual (change in TFP); factor substitution, and output elasticities. In this way we can present and contrast the effects of including energy as input to production; CD vs. CES functions; quality-adjusting production factors; and time variance.

1 Introduction

Capital (stock) and labour (number of workers) are the two factors of production traditionally considered to give rise to economic output, typically measured by Gross Domestic Product (GDP). However, not all capital and labour is equally productive, since new machines produce more output than dilapidated machines, and skilled workers are likely to generate more economic output than unskilled workers. Despite this, empirical studies of economic growth historically used unadjusted values of labour and capital as their two factors of production (e.g., Rusek, 1989; Finn, 1995; Chow &

Li, 2002). However, this picture is changing, and studies that include quality adjusted labour (e.g. adjusting work hours by educational indices) and quality adjusted capital (where the productive effect of capital stock is measured as capital services), are gaining in popularity. To date, studies that quality-adjust labour (e.g. Vouvaki & Xepapadeas, 2008; Hall & Jones, 1999) are more common than studies that quality-adjust capital (e.g. Dougherty & Jorgenson, 1996; Hájková & Hurník, 2007). Indeed, capital services is a newer field of study, and Inklaar suggests significant measurement issues remain such as the choice of the rate of return (Inklaar, 2010). Work continues in academia (e.g. Barro & Lee, 2001) and government agencies (ONS - UK Office of National Statistics, 2015) to develop more consistent datasets of capital services.

The second issue considered by this paper is the inclusion of energy as a factor of production (e.g. Kemfert & Welsch, 2000; Dissou et al., 2011). Energy-augmented specifications are rarely tested against the counterfactual (i.e., using labour and capital only), thereby limiting the interpretation of economic growth estimates. Aside from a short introduction to the rationale behind energy augmentation, we remain agnostic: we wish simply to test the effect of capital-labour versus capital-labour-energy in a production function context by assessing the effect of including energy on the Solow Residual. The hypothesis that energy is important for economic growth is supported if the Solow Residual decreases when energy is included in the production function.

Combining these two strands (testing quality-adjusted factors of production and energy augmentation), we arrive at our research question: *How does the Solow Residual change when introducing quality-adjusted values for capital, labour, and energy?* We conduct the empirical analysis for the UK and Portugal using 1960–2010 datasets, establishing key parameters including goodness-of-fit to economic growth, total factor productivity (TFP), Solow Residual, factor substitution, and output elasticities. We use both Cobb-Douglas (CD) and Constant Elasticity of Substitution (CES) functions, which facilitates a comparison between the results of the two most common aggregate production functions.

The remainder of this paper is organized as follows: Section 2 presents Background, Section 3 discusses Data and Methods, Section 4 provides Results and Discussion, and Section 5 concludes.

2 Background

We first summarise the CD and CES production functions and the Solow residual. Second, we outline the basis for the two main factors of production, namely labour and capital, and how quality-adjusted measures (human capital and capital services) are determined. Third we discuss energy augmentation: the rationale behind its inclusion as a production factor, the common (primary) energy variable and its thermodynamic-based quality-adjustment to useful work.

2.1 CD and CES aggregate production functions

2.1.1 Cobb-Douglas (CD) Production Function

The CD production function can be expressed as Equation **1**

$$y = \theta A k^{\alpha} l^{\beta};\ A \equiv \exp[\lambda(t - t_0)] \qquad \text{Equation 1}$$

where , $y \equiv Y/Y_0$, θ is a scale parameter, expected to be close to unity, exp[] is the natural exponential function, λ represents the pace of technological progress, t (time) is measured in years, $k \equiv K/K_0$, $l \equiv L/L_0$, Y (economic output) is represented by GDP, K (capital) is expressed in currency units, L (labour) is expressed in workers or work-hours/year, and the 0 subscript indicates values at initial base year[1]. Constant returns to scale are represented by the constraint $\alpha + \beta = 1$.

The capital-labour CD production function shown in Equation **1** can be augmented by energy as shown in Equation 2:

$$y = \theta A k^{\alpha} l^{\beta} e^{\gamma};\ A \equiv \exp[\lambda(t - t_0)] \qquad \text{Equation 2}$$

where $e \equiv E/E_0$, and E is in units of energy per time, typically TJ/year. The energy-augmented CD production function is often assumed to have constant returns to scale (homogeneous of degree one) for the three factors of production: $\alpha + \beta + \gamma = 1$. The term A is known as total factor productivity (TFP), and λ is the Solow residual, being the annual change in TFP.

2.1.2 Constant Elasticity of Substitution (CES) function

Many economists use the CES production function (Equation 3) to describe economic growth with capital stock (k) and labour (l) factors of production.

$$y = \gamma A[\delta_1 k^{-\rho_1} + (1 - \delta_1) l^{-\rho_1}]^{-1/\rho_1};\ A \equiv \exp[\lambda(t - t_0)] \qquad \text{Equation 3}$$

Equation 4 augments Equation 3 with energy using a $(kl)(e)$ nesting structure, as is commonly used in the literature. Equation 3 is a degenerate form of Equation 4 where $\delta \to 1$.

$$y = \gamma A \left\{ \delta [\delta_1 k^{-\rho_1} + (1 - \delta_1) l^{-\rho_1}]^{-\frac{1}{\rho_1}} + (1 - \delta) e^{-\rho} \right\}^{-\frac{1}{\rho}};\ A \equiv \exp[\lambda(t - t_0)] \qquad \text{Equation 4}$$

$$A \equiv \exp[\lambda(t - t_0)]$$

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[1] Dimensionless, indexed quantities are represented by lower-case symbols $(y, k, l, e, q, x, and\ u)$ and dimensional quantities are represented by upper-case symbols $(Y, K, L, E, Q, X, and\ U)$. Model parameters are represented by Greek letters $(\alpha, \beta, \lambda, \theta)$.

The scale parameter γ is expected to be close to unity. The fitting parameters ρ_1 and ρ indicate elasticities of substitution (σ_1 and σ). The elasticity of substitution between capital (k) and labour (l) is given by $\sigma_1 = \frac{1}{1+\rho_1}$, and the elasticity of substitution between (kl) and (e) is given by $\sigma = \frac{1}{1+\rho}$. As $\rho_1 \to 0$, $\sigma_1 \to 1$, and the embedded CES production function for k and l degenerates to the CD production function. Similarly, as $\rho \to 0$, $\sigma \to 1$, and the CES production function for (kl) and (e) degenerates to the CD production function. As $\sigma \to \infty$ ($\rho \to -1$), (kl) and (e) are perfect substitutes. As $\sigma \to 0$ ($\rho \to \infty$), (kl) and (e) are perfect complements (Leontief production function): no substitution is possible. Similarly, as $\sigma_1 \to 0$ ($\rho_1 \to \infty$), k and l are perfect complements. Figure 1 exemplifies the isoquants for each case of the two-factor CES production function – Equation 3 –, which are also resumed in Table 1:

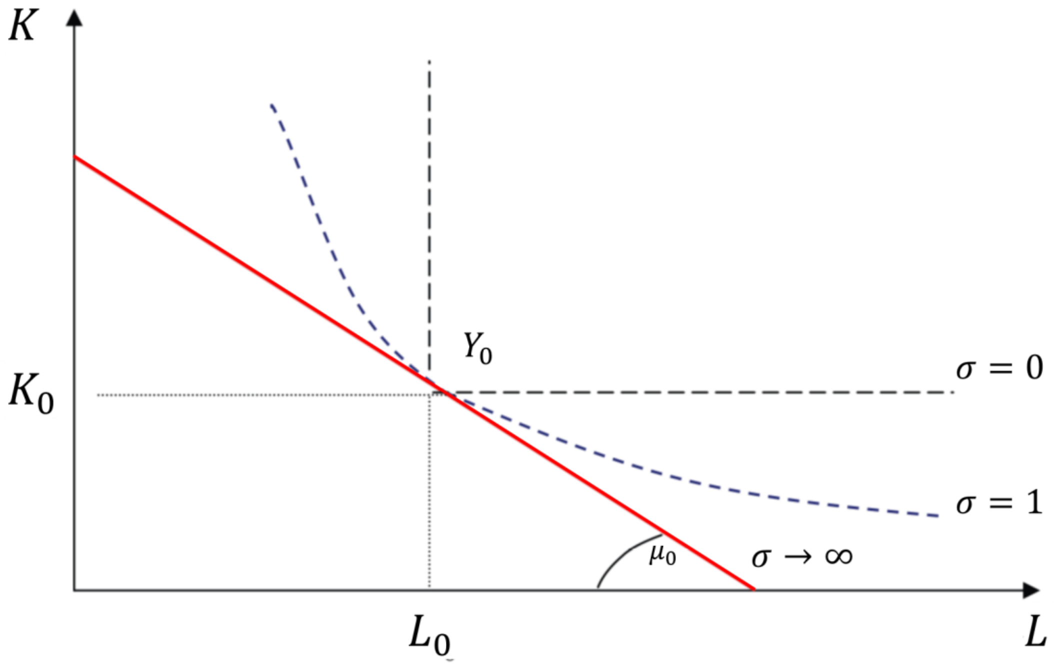

Figure 1: Two-factor CES isoquants (lines of constant output Y). K is capital and L is labour. Adapted from Klump et al. (2012).

σ_1	ρ_1	CES simplifies to...	Factors of production are...
0	∞	Leontief	Perfect complements
1	0	Cobb-Douglas	Substitutable
∞	-1	Linear	Perfect substitutes

Table 1: Two-factor CES simplifications.

The relative importance of capital (k) and labour (l) is described by δ_1, and δ describes the importance of (kl) relative to (e)

Two other nestings of the factors of production (k, l, and e) are possible with the CES model, being $le(k)$ and $ek(l)$, as shown in Equation 5 and Equation 6:

$$y = \gamma A \left\{ \delta \left[\delta_1 l^{-\rho_1} + (1-\delta_1) e^{-\rho_1} \right]^{-\frac{1}{\rho_1}} + (1-\delta) k^{-\rho} \right\}^{-\frac{1}{\rho}}; \quad A \equiv \exp[\lambda(t - t_0)] \qquad \text{Equation 5}$$

$$y = \gamma A \left\{ \delta \left[\delta_1 e^{-\rho_1} + (1-\delta_1) k^{-\rho_1} \right]^{-\frac{1}{\rho_1}} + (1-\delta) l^{-\rho} \right\}^{-\frac{1}{\rho}}; \quad A \equiv \exp[\lambda(t - t_0)] \qquad \text{Equation 6}$$

Similar to the CD model, the term A is known as total factor productivity, and λ is the Solow residual.

2.2 Production factor #1: Labour, L

2.2.1 Unadjusted labour

The simplest measure for labour inputs is the number of workers. However, this measure weighs all workers equally, regardless of whether they work part-time of full-time. Alternatively, the standard measure for labour is work hours. While this method recognizes the fact that hours worked differ from individual to individual, it does not account for differences in the productive capacity of different individuals. Across aggregate production function analysis, labour inputs are generally measured as number of workers, or number of persons engaged in economy activity (e.g. Barro & Sala-i-Martin, 1990; Hall & Jones, 1999). This is because data on employment is much more readily available than data on total hours worked, for long periods of time. However, increasing availability of historical data on average hours worked (e.g. international databases such as Penn World Table) has led to several production function analyses using this measure (e.g Van der Werf, 2008).

2.2.2 Quality-adjusted labour

By adjusting total hours worked with a labour quality index, a more representative measure of the contribution of labour to production can be obtained. The aim of quality-adjusting labour inputs is to assess the impact of growth in labour services to economic growth over time (Jorgenson, Gollop & Fraumeni, 1987). However, adjusting labour for quality, by measuring the skill level (quality) of workers, is difficult, because “skill” is a loose term that cannot be directly observed, is embodied in a variety of forms[2], and individuals’ skill levels are inherently subjective. To capture the quality of labour, it is then necessary to resort to proxies. A production-oriented definition of *human capital* – defined as an amalgam of factors such as education, experience, training, etc. – can be approximated

[2] e.g. innovation and creativity, work experience, education, etc.

by a limited number of observable characteristics, primarily the amount of formal schooling[3]. Empirical papers based on schooling include (Autor et al., 2006; Dougherty & Jorgenson, 1996; Daude, 2014).

In recent years, the Organisation for Economic Co-operation and Development (OECD) has attempted to produce comprehensive measures for the stock of human capital (Brian, 2007).

The Penn World Tables – Version 8.1 (April 13th 2015) – database features an index of income-based measured human capital that is comparable across countries and over time. For some countries, data goes back as far as 1950 (Portugal, UK), while for other economies the series start at 1970 (Hungary, Poland) or as late as 1990 (Ukraine). All series have data up to 2011. This human capital index, h, is constructed following the broader literature, namely Hall & Jones (1999), as a function of the average years of schooling, which are drawned from the international database compiled by Robert Barro and Jong-Wha Lee[4]. The methodology adopted by the Penn World Table database and applied in our empirical analysis is described in Appendix A.

2.3 Production factor #2: Capital, K

2.3.1 Unadjusted capital (stock)

Rigorous measurement of capital inputs to production processes is essential to productivity and economic growth analysis, especially when it comes to properly quantifying the sources of growth in terms of rising TFP versus factor accumulation. However, the measurement of capital inputs forms a complex conceptual problem, on which the relevant literature has extensively focused[5]. Most productivity analyses use the stocks of assets to measure capital inputs. The most frequently available measures for capital stocks are the gross capital stock (GCS) and net capital stock (NCS).

GCS is described as the sum of past investments (valued at current purchasers' prices for new assets of the same type, regardless of the asset's age), corrected only for a retirement pattern. GCS measures are most commonly obtained through the Perpetual Inventory Method (PIM)[6], which

[3] Given its broad coverage of countries and years, the average years of schooling remains the most useful measure of human capital. However, the quality of education, as reflected in internationally comparable test scores, is also increasingly flagged as an important dimension of human capital (Hanushek & Woessman, 2012; Caselli, 2005).

[4] Available online at http://www.barrolee.com/. Data used corresponds to average years of education of population aged 15 and older (15+).

[5] Hicks (1974) present an overview of some aspects of the capital controversy, both among classical and modern economists. Also see discussions by Denison (1957) and Griliches & Jorgenson (1966).

[6] Gross capital stock can also be estimated directly, based on data from insurance records, book values or direct data collection.

produces an estimate of the stock of fixed assets at a given moment by accumulating past capital formation and deducting retired or written off assets.

Net capital stock (NCS), or wealth stock, measures the market value of capital assets. It corresponds to GCS after correcting for loss in value due to ageing, besides retirement of assets (usually by deducting accumulated consumption of fixed capital – CFC). Both measures are commonly adopted in the literature on production function analyses (e.g. Kemfert, 1998; Smulders & De Nooij, 2003) since data is usually equally available. The European Commission's annual macro-economic database (AMECO) publishes NCS series and CFC series that allow for estimation of GCS[7].

2.3.2 Adjusted capital: capital services

One major concern is that GCS and NCS do not measure the productive efficiency of assets i.e. the flow of productive services from the cumulative stock of past investments – *capital services*. It is based on the economic theory of production, and as a concept relates back to the work by Jorgenson & Griliches (1967), the first to develop aggregate capital service measures that account for the heterogeneity of assets. Further developments were made mainly in the productivity literature, such as Hulten (1996), and Diewert (2001).

Conceptually, capital services reflect a physical quantity, not to be confused with the value or price concept of capital[8]. A direct analogy can be made regarding the service measures of capital and labour inputs: in the same way that employees hired for a certain period of time can be seen as stocks of human capital and therefore repositories of labour services, so can physical assets rented or purchased by a firm be carriers of capital services. Examples of the use of capital services in empirical production function papers include Schreyer (2004) and Hájková & Hurník (2007), However, whilst the desirability of capital services (as opposed to capital stock) was noted long ago by Solow (Solow, 1957), measurement issues remain an important caveat to their use, as highlighted by Inklaar (Inklaar, 2010).

2.4 Production factor #3: Energy, E

2.4.1 Energy as a production factor candidate

Widespread evidence supports linkages between energy use and economic growth (e.g. Kalimeris et al., 2014; Bruns et al., 2014), and the subsequent view that energy has been an extremely important factor for economic growth in the last decades (Kümmel et al., 2010; Ayres & Warr, 2005). However,

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[7] Series available at http://ec.europa.eu/economy_finance/ameco/user/serie/ResultSerie.cfm

[8] For example, an office building: among the service flows one includes the protection against rain, as well as the comfort and storage services provided to personnel during a given period.

standard economic growth theory assumes only two factors of production due to: a) an accounting identity defining output as the sum of payments to capital and labour (SNA, 2008); b) the empirical observation that these cost-shares are constant in the long-run (Kaldor, 1961); c) an income allocation theorem stating that output elasticities are proportional to cost-shares (Solow, 1957). The small cost-share attributed to energy therefore suggests it should not be considered an important contributor to productivity (Denison, 1979). However, it is clear that neither capital nor labour can function without energy. A flow of energy capable of doing work is as essential for production, and it should be regarded as a production factor (Ayres et al., 2013). In the event, our agnostic approach is simply to test the effect of including energy as a variable on the outturn Solow Residual.

2.4.2 Unadjusted (primary) energy

Energy comes in many forms (e.g. chemical, nuclear, thermal, electrical, and mechanical), and all are inputs to economic production. A variety of methods have been adopted in the literature to aggregate energy inputs and, while none has received universal acceptance, the simplest and most widespread method used in economics and ecology studies is the basic heat equivalents approach. This method consists of adding up individual energy inputs according to their thermal equivalents (in BTU or joule units). The heat equivalents approach is both simple and well-defined, and data is readily available – for example the International Energy Agency publishes energy balances and time series on primary and final energy in thermal equivalents. These datasets are the ones generally used in production function analysis (besides national official statistics) (e.g. Van der Werf, 2008).

2.4.3 Quality-adjusted energy (useful work)

The weakness in unadjusted energy is that it considers only one attribute of a given fuel - heat equivalents – and so ignores qualitative differences among energy types – i.e. the context in which the fuel is used[9]. Similar to both capital and labour, it is services provided by thermal energy inputs that are economically productive.

Two main options for quality-adjusting energy exist. The first is a method for aggregating energy flows based on prices, where the higher price paid for an energy source (e.g. electricity) reflects its higher quality as an energy vector. A fuller description is given by (Stern, 2010). The second method – and the one used in this paper – is based on thermodynamic principles: utilizing the concept of available energy (i.e. exergy), and the ability of exergy to provide "useful work" at the point of use[10]. The

[9] It cannot thus explain why a thermal equivalent of oil is, in many tasks, more useful than a thermal equivalent of coal, for example.

[10] Useful exergy and useful work are interchangeable terms defining the same concept. Throughout the rest of this paper only the nomenclature "useful work" will be used.

concept of exergy is defined as "*the maximum amount of physical work that can be performed by a system as it reaches thermodynamic equilibrium with its surroundings (environment), reversibly*" (Moran et al., 2010). Exergy constitutes a measure for energy quality, because it accounts for the "usefulness" of energy against reference environmental conditions. The exergy method for quality-adjustment has been extensively presented in the literature as a good approach to economic and sustainability assessments of energy flows (Ayres et al., 2007; Dincer et al., 2007; Ertesvag et al., 2000; Hammond et al., 2001).

There is an energy conversion chain from raw energy carriers to energy services at the point of use in the economy, as shown in Figure 2. (E.g., coal → heat → steam → spinning turbine shaft → generator → electricity at plant → electricity at point of use → spinning shaft → machine motion → parts produced (the energy service).) At each stage through the energy chain, conversion losses accumulate through inefficiencies. Indeed, the last stage (conversion to an energy service) involves losses at final devices, such as lamps, motors, etc.). Useful work situates the analysis at the level of satisfied energy needs, as close as possible to energy services while still maintaining energy units of measure.

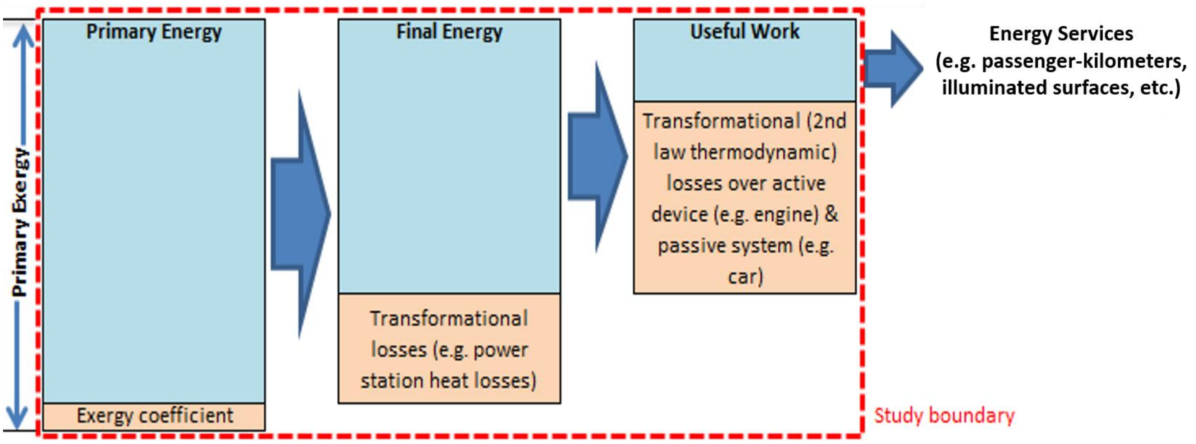

Figure 2: Conceptual diagram of exergy to useful work (adapted from Brockway et al, 2015)

Since the first national-scale study by Reistad (1975), more national-scale datasets of useful work are emerging (e.g. Warr et al., 2010; Serrenho et al., 2015; Brockway et al., 2014). The inclusion of useful work as a production factor has been rarely examined to date (e.g. Ayres & Voudouris, 2014; Ayres & Warr, 2005), making it a good candidate for inclusion in this current paper.

3 Data and Parameter Estimation

3.1 Data

Our analysis focuses on two countries: Portugal and the United Kingdom. The choice of these two economies is justified by the availability of datasets for these countries regarding capital services data (Oulton & Wallis, 2015; Da Silva & Lains, 2013) and useful exergy (Serrenho et al., 2015; Brockway et al., 2014). The period of analysis extends from 1960 to 2009, because (a) it offers a lengthy 50-year time-series, (b) official statistics commonly begin in 1960, and (c) it avoids the economic fluctuations of WWI and WWII.

Figure 3 shows all historical data.

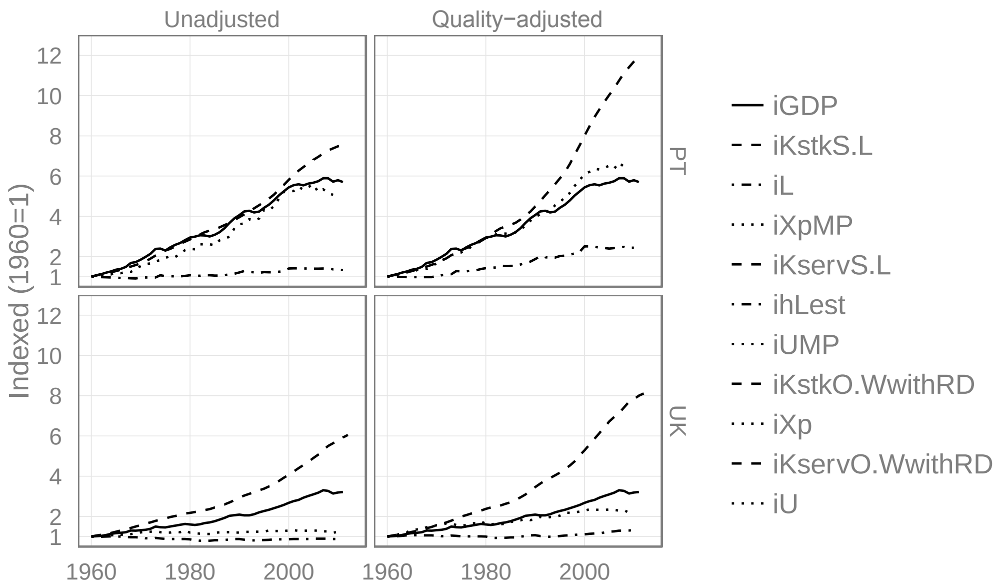

Figure 3: Historical data. Series indexed to 1960 = 1. GDP is represented as a full line, capital inputs (stocks and services) are dashed lines, labour inputs (with and without human capital) are dot-dash-dot lines, and energy inputs (primary and useful exergy) are dotted lines.

3.1.1 Economic Output

Gross domestic product (GDP) is the most widely adopted measure for economic output in productivity analysis, and is the standard indicator in national accounts and international databases to measure the economic performance of a given country or region. Throughout this analysis we will also adopt GDP (in constant prices $2005 US) as the appropriate measure for economic output. Time series of GDP are obtained from Penn World Tables (Version 8.1).

3.1.2 Labour inputs

For unadjusted labour, we use data on total number of hours worked, obtained by multiplying data on average hours worked per individual by the number of persons engaged each year, both obtained from the Penn World Tables (Version 8.1).

For quality-adjusted labour, we adopt the methodology from the Penn World Tables (Version 8.1) to compute a human capital index, but adopt the most recent data on average years of schooling from the Barro-Lee database (Version 2.0).

The methodology adopted from the Penn World Tables for computing a human capital index is applied to both countries: Portugal and the UK. The method is described in more detail in Appendix A. This human capital index is multiplied by the annual total number of hours worked by engaged persons in each country[11] – corresponding to unadjusted labour inputs, L – in order to construct a measure for quality-adjusted labour inputs, $h \cdot L$. Figure 4 and Figure 5 compare the unadjusted labour (iL) and human-capital adjusted labour inputs (ihLest) for Portugal and the UK, respectively.[12]

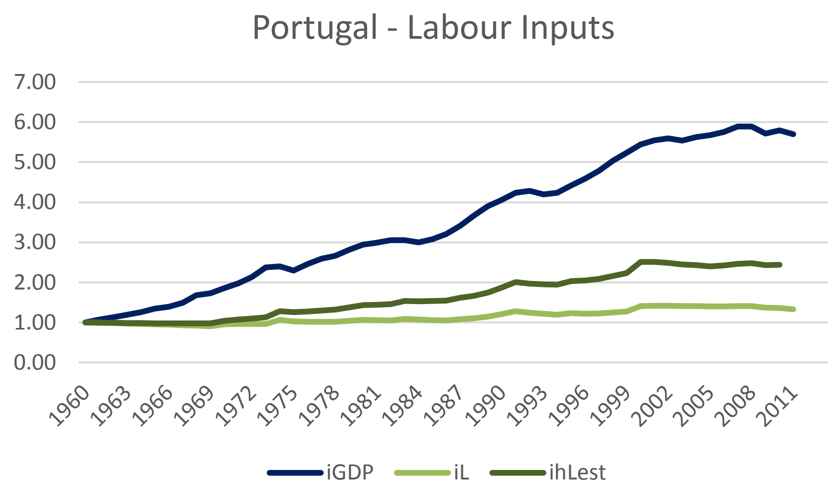

Figure 4: Unadjusted (iL) and human-capital adjusted (ihLest) labour inputs estimates for Portugal (1960-2010), derived using Penn World Tables methodology and data, as well as average years of schooling data obtained from the Barro-Lee database. Series compared with gross domestic product (iGDP) for the same period. All series indexed to 1960 = 1.

[11] The total hours worked by persons engaged is in turn determined by multiplying the number of persons engaged by the average hours worked per person engaged (all data available at the Penn World Tables).

[12] “i" prefixes on variable names indicate “indexed” (1960 = 1) quantities.

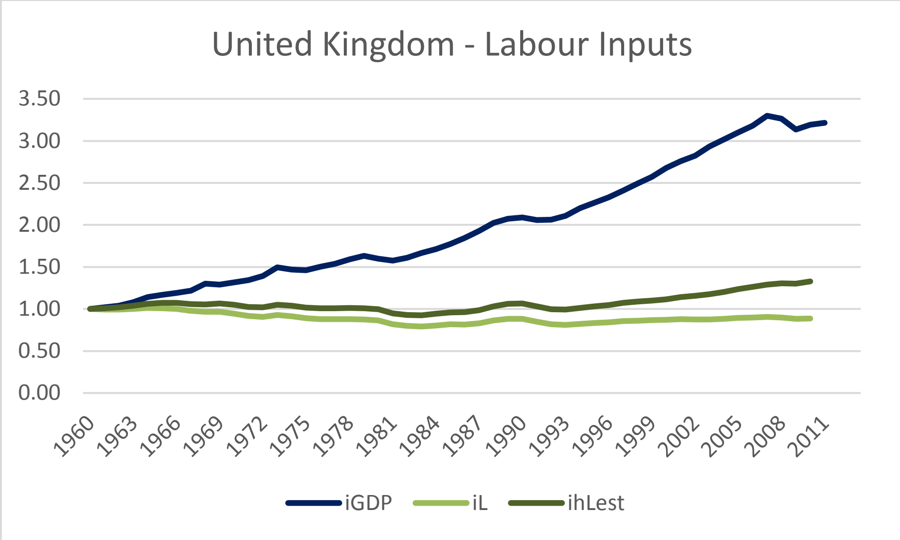

Figure 5: Unadjusted (iL) and human-capital adjusted (ihLest) labour inputs estimates for the UK (1960-2010), derived using Penn World Tables methodology and data, as well as average years of schooling data obtained from the Barro-Lee database. Series compared with gross domestic product (iGDP) for the same period. All series indexed to 1960 = 1.

Regardless of quality adjustments, labour inputs exhibit overall lower growth than economic output (GDP). On the other hand, quality-adjusted labour inputs show an overall higher growth than unadjusted labour inputs.

3.1.3 Capital Inputs

3.1.3.1 Portugal

Available capital stock estimates for Portugal generally refer only to partial periods within the 20th century (Santos, 1984; Pina & St. Aubyn, 2005; Teixeira & Fortuna, 2010; Da Silva, 2010), and are not directly comparable, due to significant differences in assumptions and estimation procedures. Da Silva (2010) provides capital services' estimates for an extended period of time (1977-2003) in Portuguese industries, and subsequently Silva & Lains (2013) provide an integrated and consistent approach on the estimation of capital stocks and flows for the whole period between 1910 and 2011. We therefore adopt in our analysis the capital stocks and services estimates from this latter study, for the years 1960-2009. The detailed methodology is described in Appendix A. Figure 6 illustrates the differences between these two series, and compares them with Portuguese gross domestic product (GDP) for the same time period.

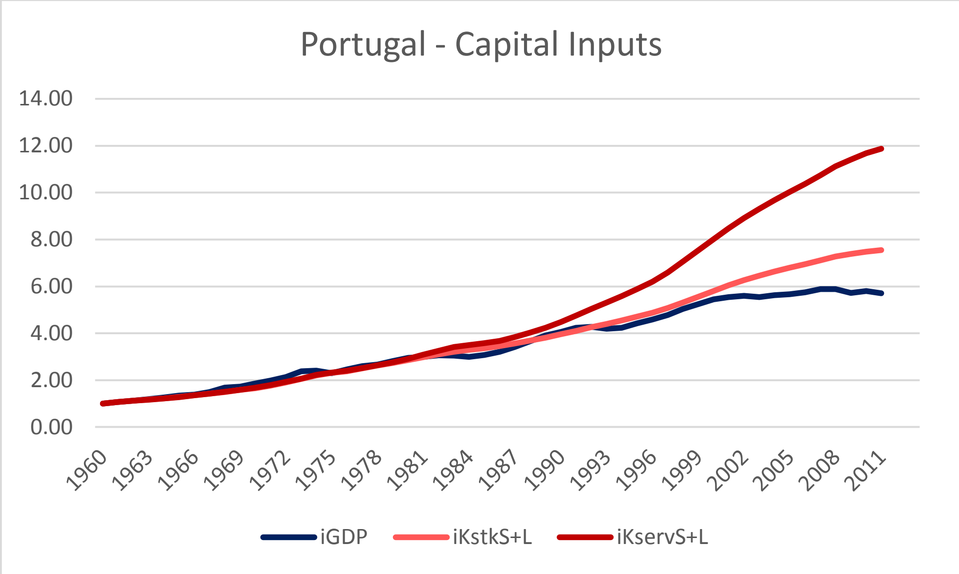

Figure 6: Capital stock (iKstkS+L) and capital services (iKservS+L) estimates from Silva & Lains (2013) for the Portuguese economy. Series compared with Portuguese GDP (iGDP) for the same period. All series indexed to 1960 = 1.

3.1.3.2 United Kingdom

Estimates for the stocks and services of capital in the United Kingdom are much more organized and abundant than what is available for the Portuguese case. The methodologies adopted are also considerably less simplistic in its assumptions and scope. Official estimates of capital stocks in the UK relate back to the work of Redfern (1955) and Dean & Irwin (1964). The UK's Office for National Statistics (ONS) latest capital stock estimates were released in July 2014, covering only the period from 1997 to 2012. Unlike Portugal, the UK's ONS has also regularly produced "experimental" estimates of capital services since 2005. However, the assumptions employed in these sets are not consistent with those used by the ONS for capital stocks[13], and the data series in general should be viewed with caution.

Realizing the need for a more consistent framework for the estimation of capital stocks and services in the UK, Oulton & Wallis (2015) have developed a set of estimates for both these variables following the recommendations of the OECD's manual Measuring Capital (Schreyer, 2009). These estimates make use of up-to-date data, making proper allowance for ICT (information and communications technology) assets and taking into account the recent Eurostat requirement that R&D should be incorporated in National Accounts as a form of investment. For the United Kingdom, we therefore adopt the capital stocks and services estimates presented in Oulton & Wallis (2015).

[13] The depreciation assumptions differ, as do some of the price indices (e.g. ICT assets).

Figure 7 compares the stock and service capital estimates from Oulton & Wallis with GDP for the United Kingdom in the same period (1960-2010). Oulton & Wallis (2015) compute different series for both stocks and services, taking different assumptions regarding the inclusion or not of R&D assets, the use of an ex-post or hybrid method for estimating the rates of return, or the choice of aggregate index between Törnqvist and Laspeyres. For the purposes of our analysis, we consider only the series computed for which the rate of return is estimated using an ex-post approach, R&D assets are included, and aggregation is done with a Törnqvist index. Despite the considerable differences in assumptions between Portugal and UK capital estimates, this selection of series for the UK allows for a more comparable set of data between the two countries.

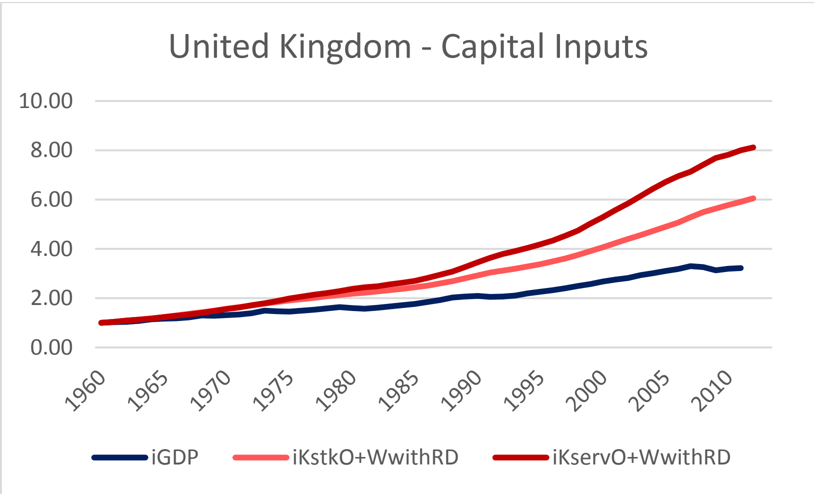

Figure 7: Capital stock (iKstkO+WwithRD) and capital services (iKservO+WwithRD) estimates from Oulton & Wallis (2015) for the United Kingdom, including R&D assets. Series compared with UK GDP (iGDP) for the same period. All series indexed to 1960 = 1.

3.1.4 Energy Inputs

In our analysis, we will adopt exergy measures for the primary and useful stages of the energy flow, as appropriate energy inputs. A detailed methodology for useful work is presented in Appendix A.

3.1.4.1 Portugal

Taking into account the improved methodology adopted by Palma (2014) for useful exergy accounting in the Portuguese economy, we will use these datasets for primary exergy (X_P, multiplying primary energy by the exergy content factors) and useful exergy (X_U) in our analysis. Figure 8 compares these variables for Portugal, 1960-2009.

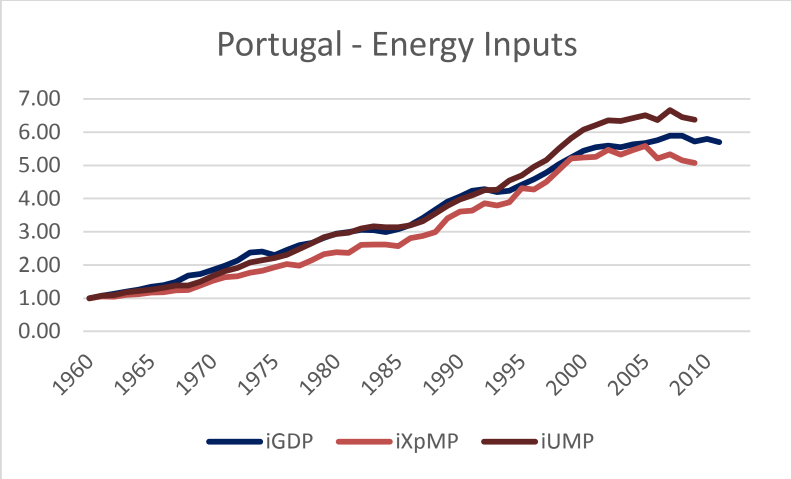

Figure 8: Palma's (2014) primary exergy (iXpMP) and useful exergy (iUMP) estimates for the Portuguese economy, 1960-2009. Comparison with economic output for the same period (iGDP). All time series indexed to 1960 = 1.

3.1.4.2 United Kingdom

Recently, Brockway et al. (2014) have conducted a national exergy efficiency analysis for the UK, for the period 1960-2010. This effort also builds on the methodology introduced by Warr et al. (2010), and on the changes introduced by Serrenho et al. (2015), regarding the standardization of primary energy mapping to useful exergy categories based on IEA datasets. We therefore adopt this dataset for primary exergy and useful work for the UK, as shown in Figure 9:

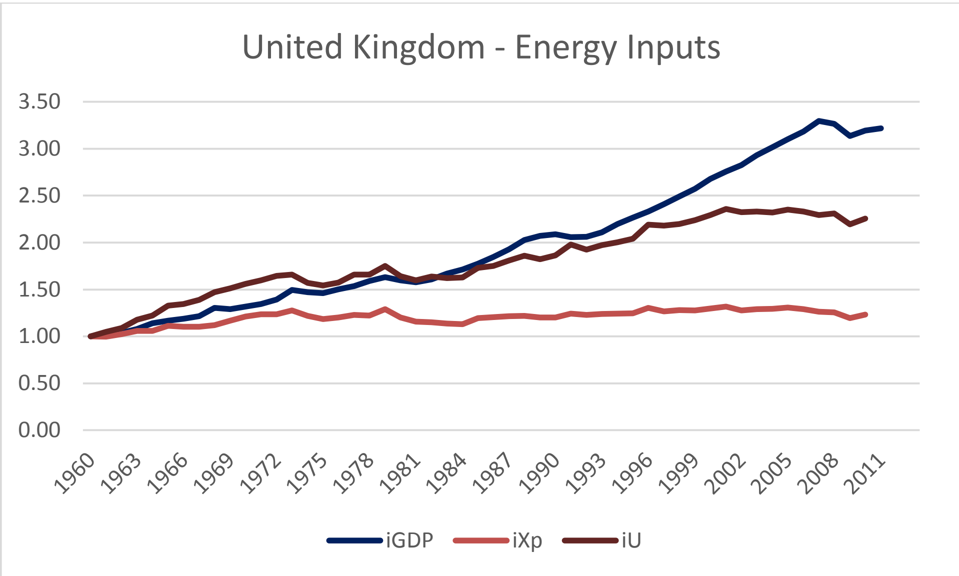

Figure 9: Brockway et al. (2014) primary exergy (iXp) and useful exergy (iU) estimates for the United Kingdom, 1960-2009. Comparison with economic output for the same period (iGDP). All time series indexed to 1960 = 1.

3.2 Parameter Estimation

3.2.1 Overview

To estimate the parameters (Greek letters) in Equations 1, 2, 3, 4, 5 and 6, time series for indexed factors of production (unadjusted and quality-adjusted, without and with energy) are regressed against economic output (GDP) for both CD and CES production functions. We assume constant-returns-to-scale and positive but decreasing or constant returns to inputs. By definition, the CD function has unitary elasticity of substitution, and constant output elasticities for any input combination, allowing asymptotically for complete substitution. It does not allow a distinction between factor-augmenting innovations. CES functions assume a more general, not necessarily unity, constant elasticity of substitution. Perfect substitutes (linear), perfect complements (Leontief) and CD functions are degenerate cases of the CES function. For three factors of production – Capital (k), labour (l) and energy (e) – we test the three possible types of nested structure for the CES: $(kl)e$, $(le)k$, and $(ek)l$. Models are compared among themselves and with analogous formulations without any energy inputs, in terms of goodness-of-fit, TFP, factor substitution and output elasticities. We obtained parameter estimates by applying the method of least-squares to log-transformed data using the R (R Core Team, 2012) functions `lm` (for CD models) and `cesEst` (Henningsen & Henningsen, 2011) (for CES models).

3.2.2 CD model

The CD model without energy is given by Equation **1**, which can be re-parameterized as Equation 7:

$$y = \theta \exp[\lambda(t - t_0)] k^{\alpha} l^{1-\alpha} \qquad \text{Equation 7}$$

To ensure $\alpha + \beta = 1$ for constant returns to scale. θ, λ, and α were estimated in log-transform space[14] as shown in Equation 8:

$$\ln y = \ln \theta + \lambda(t - t_0) + \alpha \ln k + (1 - \alpha) \ln l \qquad \text{Equation 8}$$

If the estimated value for α was found outside the interval $[0, 1]$, we set α to its boundary value and re-estimated λ. The value of β was found with $\beta = 1 - \alpha$. To estimate the parameters θ, λ, α, β, and γ in the energy-augmented CD model, Equation 2 was reparameterized as Equation 9, ensuring that $\alpha + \beta + \gamma = 1$, thereby providing constant returns to scale.

$$y = \theta \exp[\lambda(t - t_0)] k^{\alpha} l^{\beta} e^{1-\alpha-\beta} \qquad \text{Equation 9}$$

[14] For detailed steps on how to estimate the Solow residual using a Cobb-Douglas production function, see Appendix B.

Again, values of λ, θ, α, and β were estimated in log-transform space as shown in Equation 10:

$$\ln y = \ln \theta + \lambda(t - t_0) + \alpha \ln k + \beta \ln l + (1 - \alpha - \beta) \ln e \qquad \text{Equation 10}$$

If the fitted value for α or β fell outside the interval $[0,1]$, we fit along all boundaries ($\alpha = 1$, $\beta = 1$, $\gamma = 1$, $\alpha = 0$, $\beta = 0$, and $\gamma = 0$) and chose the boundary fit with minimum sum of squared errors as the winning model. γ was calculated as $\gamma = 1 - \alpha - \beta$.

3.2.3 CES model

The CES model without and with energy is given by Equations 3, 4, 5 and 6. The R (R Core Team, 2012) package `micEconCES` (Henningsen & Henningsen, 2011) was used to estimate parameters γ, λ, δ_1, δ, ρ_1, and ρ. The `cesEst` function of `micEconCES` provides several algorithm options for parameter estimation. The default algorithm (Levenberg-Marquardt) does not respect parameter constraints (see Section 2.1.2) and, in our testing, nearly always violated them, often returning negative values for elasticity of substitution parameters σ_1 and σ. Thus, we used the two fitting algorithms available in `cesEst` that respect coefficient constraints: PORT and L-BFGS-B.

Our CES parameter estimation algorithm starts with an eleven-value grid search in ρ_1 and ρ (9, 2, 1, 0.43, 0.25, 0.1, -0.1, -0.5, -0.75, -0.9, -0.99), corresponding to σ_1 and σ values of 0.1, 0.33, 0.5, 0.7, 0.8, 0.9, 1.11, 2, 4, 10, and 100, respectively. During the grid search, values of ρ_1 and ρ are fixed, and values of γ, λ, δ_1, and δ are estimated by gradient search with the PORT and L-BFGS-B algorithms. In all, 121 gradient searches in γ, λ, δ_1, and δ at grid points representing all combinations of ρ_1 and ρ are attempted. During the grid search portion of our algorithm, starting values for the free parameters are $\lambda = 0.015/year$, $\delta_1 = 0.5$, $\delta = 0.5$, and γ is set by the `cesEst` function to a value such that the mean of the residuals is zero.

Next, a gradient search (using both PORT and L-BFGS-B) is attempted wherein all fitting parameters (γ, λ, δ_1, δ, ρ_1, and ρ) are allowed to float. The starting values for fitting parameters are taken from the grid search point that provided the lowest sum of squared errors (sse).

In addition to the above trials, and in a manner similar to our approach to the CD model, we also fit along all possible CES boundaries. Boundaries are given by all combinations of the following: $\delta_1 = 0$ or 1, $\delta = 0$, or 1, $\sigma_1 = 0$ or ∞, and $\sigma = 0$ or ∞. There are 20 degenerate equations found along parameter boundaries. Each degenerate equation provides a boundary model.

The model with lowest sse of all above trials is deemed the winning (i.e., best) model.

3.2.4 Statistical checks

Pending.. Initial results and findings to be presented at ESEE 2015

4 Results and Discussion

4.1 Factors of Production

4.1.1 Comparison of capital inputs between Portugal and United Kingdom

For both economies considered in this analysis, Portugal and the United Kingdom, capital services estimates exhibit overall higher growth rates than comparable capital stocks estimates, as shown earlier in Figure 6 and Figure 7, particularly in the later period of analysis. In fact, in both cases, capital services have grown consistently faster than capital stocks over the last 50 years.

In Portugal, capital services estimates assume higher average growth rates, due to the exclusion of residential capital and to the different method applied in the aggregation of assets. More precisely, the changes observed in the composition of capital, with the increase in the relative importance of short-lived assets, gave rise to an increasing gap between growth rates of capital services and stocks. By the end of the period, capital services are approximately 57% higher than capital stocks.

For the UK, Oulton & Wallis (2015) have noted that capital services estimates grow more rapidly than stocks over the whole studied period and sub-periods. The difference in overall growth rates accounted by these authors is approximately 0.4 per cent a year. By the end of the period, capital services are approximately 34% higher than capital stocks. Capital services exhibit a more rapid growth between 1970 up to around the mid-2000s. After 2008, and the economic crisis, capital services growth has slowed down and stocks grew faster than services. Both capital stocks and services estimates in both countries grow on average faster than economic output for the overall period.

4.1.2 Comparison of labour inputs between Portugal and the UK

With or without quality adjustments brought by the inclusion of a human capital index, labour inputs exhibit overall lower growth than economic output (GDP). On the other hand, quality-adjusted labour inputs show an overall higher growth than unadjusted labour inputs.

This is especially true for Portugal, where inclusion of a human capital index raises labour growth by a significant amount (approximately 50% by the end of the period), when compared with the UK. This is due to the very low levels of education in Portugal in the beginning of the considered period. Portugal's literacy rate by the 1940s and early 1950s was low for North American and Western European standards at the time. Only after 1960 was public education made available for all children between ages six and twelve. This was accompanied by the expansion of a network of industrial and commercial schools and the foundation of new state-run universities up until the Carnation Revolution in 1974. After 1974 the number of basic and secondary schools, as well as of higher education institutions, increased until the end of the century. Throughout the entire period, the average number of hours worked also increased on average. One can observe a peak in both unadjusted and quality-adjusted labour inputs in 1974-75, due respectively to the expansion of education opportunities and the return of many individuals living abroad (exiled, or living and fighting in the Portuguese colonies in Africa) to the main country and its workforce.

On the other hand, in the UK, an overall decrease in the average level of hours worked is observed throughout this period. According to OECD reports, despite having a higher percentage of employees that work very long hours (about 12%, when the OECD average is 9%), people in the UK work 1654 hours a year**,** less than the OECD average of 1765 hours. It is likely that the introduction of Working Time Regulations had some effect on decreasing working hours in the UK. These regulations were introduced in 1998 and govern the time that people in the UK may work. The Regulations apply to all workers (not just employees) and stipulate minimum rest breaks, daily rest, weekly rest and the maximum average working week. Namely, they set a maximum of 48 hours per week (which may be opted-out), a mandatory right to a paid annual leave of at least 28 days, and a minimum period of rest of 20 minutes in any 6-hour shift.

The human capital index in the UK grows slower than for Portugal, because the levels of education (i.e. years of schooling) in the UK were higher in 1960 than for Portugal. Still, accounting for human capital brings the quality-adjusted labour inputs to a situation of positive overall growth throughout the considered time period.

4.1.3 Comparison of Energy Inputs between Portugal and the UK

The primary/useful exergy estimates for Portugal and the UK show a very different picture, due to each countries' characteristics when it comes to energy use. For Portugal, primary and useful exergy estimates closely follow economic output trends, with primary exergy slightly below for the whole period, and useful exergy slightly above after 1992-93. As for the UK, primary exergy estimates are almost flat when compared with economic output growth. Useful exergy is above GDP for the earlier decades 1960-80, and below it for the rest of the period, with slightly higher overall growth than primary exergy.

The United Kingdom's average exergy efficiency in the considered time period rose from 9% to 15%, with gains in all three main sectors: heat rose from 8% to 12% (due to significant gains in task-level efficiencies); electricity rose from 8% to 14% (largely due to a rise in electricity generation efficiency from 30% to 43%); and mechanical drive rose from 11% to 21% (due to dieselisation and increases in fuel economy). This large rise in exergy efficiency explains the almost doubling of useful exergy estimates between 1960 and 2009. Useful exergy in the United Kingdom is split fairly between direct heat uses (30%), direct mechanical work (32%), and electricity end uses (38%). Manual mechanical work accounts for only 0.03% total end useful exergy, which reflects the UK's mature industrialised economy.

For Portugal, absolute figures of primary exergy consumption rise significantly from the 1960-decade onwards, as a consequence of vast industrialisation programs known as the second industrialisation era in Portugal. The transition from biomass products to fossil fuels is also more relevant in the Portuguese case than for the UK, in the considered time period. Useful exergy in Portugal also increased substantially from 1960 to 2009, a 6-fold increase. The structure of useful exergy consumption also changes much more dramatically than in the UK: the economy evolved to a higher dependency on mechanical drive services, while other electric uses and higher temperature heat uses also gained importance. These changes happened as a consequence of the industrialization of the country and mobility needs that led to increases in transport sector energy uses.

4.2 Output results

Pending.. Initial results and findings to be presented at ESEE 2015

5 Conclusions

Pending.. Initial results and findings to be presented at ESEE 2015

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Appendix A – detailed method of quality adjusted variables

A1 – Labour data

Given a general Cobb-Douglas production function of the form $Y = A \cdot K^{\alpha} \cdot (h \cdot L)^{\beta}$, where quality-adjusted labour inputs are represented as $h \cdot L$, perfect competition in factor and goods markets implies that the average wage of a worker with s years of education is proportional to his human capital. Since the wage-schooling relationship is widely thought to be log-linear, this calls for a log-linear relation between h and s as well (Caselli, 2005):

$$h = \exp[\varphi(s)] \quad \text{(A1.1)}$$

Where $\varphi(s)$ is a function of the years of schooling s. Following Caselli (2005) and Psacharopoulos (2004), there is evidence that earlier years of education have a higher return (evidenced by higher wages) than later years. This finding is based on Mincerian cross-country wage regressions. The function $\varphi(s)$ is then chosen to be a piecewise linear with slope defined according to a range of average years of schooling. The rates of return are based on Psacharopoulos (2004):

$$\varphi(s) = \begin{cases} 0.134 \cdot s, \;\; if\ s \leq 4, \\ 0.134 \cdot 4 + 0.101 \cdot s, \;\; if\ 4 \leq s \leq 8, \\ 0.134 \cdot 4 + 0.101 \cdot 4 + 0.068 \cdot (s - 8), \;\; if\ s \geq 8 \end{cases} \quad \text{(A1.2)}$$

Although the human capital index h could be estimated by adopting a constant $\varphi(s)$ function, international data on education-wage profiles (Psacharapoulos, 1994) suggests that in Sub-Saharan Africa – which has the lowest levels of education – the returns to one extra year of schooling are in the order of 13.4% while the World average is in the order of 10.1% and the OECD average in the order of 6.8%. Hence, Hall & Jones' (1999) function tries to reconcile the log-linearity at the country level with the convexity across countries.

A2 – capital data

A2.1 portugal

The methodology adopted by Silva & Lains (2013) is a step-by-step approach in which the first and most crucial task regards the construction of fully integrated investment (or gross fixed capital formation – GFCF) series. Official national accounts provide GFCF series by asset type (machinery & equipment, transport equipment, dwellings, other buildings & structures, other investment) and corresponding price indices between 1953 and 1995 (Bank of Portugal). The Portuguese Statistical Office (INE) also provides estimates on the same variables between 1977 and 2011. Both sources are compatible with the requirements stipulated by the European System of National and Regional

Accounts (SEC 95), and Silva & Lains (2013) integrates them by applying backwards the growth rates implicit in the earlier temporal series.

After GFCF series and price indices of investment goods are obtained, and consistency checks are performed, Silva & Lains (2013) estimate capital stocks using the Perpetual Inventory Method (PIM). This method produces an estimate of the stock of fixed assets in existence at a certain moment in time by accumulating past capital formation (GFCF) and deducting assets which are retired or written off. Besides investment series by assets type and producer price indices to deflate investment expenditure series, PIM requires assumptions on the depreciation of each asset type, and an initial benchmark for the respective stocks of capital.

Depreciation rates for each asset type are set by Silva & Lains (2013) using the method of declining balances suggested by Hulten & Wykoff (1996), under which the depreciation rate of asset i is computed as $\delta^i = R/\overline{T}_i$, where R is an estimated declining balance rate and $\overline{T}_i$ is the average service life of the asset. Silva & Lains (2013) set a declining balance rate of 1.65 for machinery & equipment, and 0.91 for structures[15]. Service lives assumptions are based on previous historical studies on capital formation, along with recent evidence on the Portuguese case (Silva, 2010). Different service lives are assumed in different sub-periods, considering shorter assets' lives in the more recent decades (1960 onwards).

Initial capital stocks for the beginning of the time period considered by Silva & Lains (2013) are constructed following the steady-state approach widely used in the literature (e.g. Ohanian & Wright, 2010; Prados & Rosés, 2010; Kamps, 2006). Assuming a geometric depreciation, the growth rate of the capital stock of asset i can be expressed as

$$g_{i,t} = \frac{S_{i,t+1} - S_{i,t}}{S_{i,t}} = \frac{I_{i,t}}{S_{i,t}} - \delta \qquad \text{(A2.1.1)}$$

Where $S_{i,t}$ and $I_{i,t}$ denote the capital stock and investment in asset i in period t, respectively, with δ being the depreciation rate. Thus, the capital stock of asset i at the beginning of period t can be computed as

$$S_{i,t} = \frac{I_{i,t}}{(\delta + g_{i,t})} \quad \text{(A2.1.2)}$$

As the growth rate of the stock of capital is not known, an assumption about its magnitude is required. Silva & Lains (2013) set the rate of increase of the capital stock for each asset to the steady state rate implied by the first decade of data, assuming that investment growth rates pre-1910 were similar to those of earlier years for which information is available. Given the volatility of investment figures, Silva & Lains (2013) also use the average value of investment between 1910 and 1912, rather than the 1910 value.

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[15] Specifically: 0.91 for asset types Dwellings and Other Buildings & Structures; 1.65 for asset types Transport Equipment, Machinery & Equipment and Other Investment.

After computation of capital stocks for each asset, the volume index of capital services is derived. The method followed by Silva & Lains (2013) is the one pioneered by the Bureau of Labour Statistics (BLS). Capital stocks for each type of asset are aggregated to obtain overall measures of capital services, considering the user costs of capital as the appropriate weights. These user costs reflect the marginal productivity of the different assets under the usual assumptions of competitive markets. Specifically, user costs ($\mu_{i,t}$) measure the cost of financing the asset, corresponding to the sum of depreciation ($d_{i,t}$) and the nominal cost of financial capital ($r_{i,t}$), minus the nominal capital gain (or loss) from holding the asset for each accounting period ($p_{i,t} - p_{i,t-1}$):

$$\mu_{i,t} = r_{i,t}p_{i,t-1} + d_{i,t}p_{i,t} - \left(p_{i,t} - p_{i,t-1}\right) \quad \text{(A2.1.3)}$$

After user costs have been derived Silva & Lains (2013) combine the stocks of each asset type to obtain volume indices of capital services, using a Törnqvist index:

$$\ln\left[\frac{K_t}{K_{t.1}}\right] = \sum_i \bar{v}_i \ln[S_{i,t}/S_{i,t-1}] \text{(A2.1.4)}$$

Where $S_{i,t}$ represent, as before, the estimates of capital stock for different asset types i at time t, and $\bar{v}_i = 0.5(v_{i,t} + \bar{v}_{i,t-1})$, with:

$$v_{i,t} = \frac{\mu_{i,t}S_{i,t}}{\sum_i \mu_{i,t}S_{i,t}} \quad \text{(A2.1.5)}$$

$$g_{i,t} = \frac{S_{i,t+1} - S_{i,t}}{S_{i,t}} = \frac{I_{i,t}}{S_{i,t}} - \delta \quad \text{(A2.1.6)}$$

$$S_{i,t} = \frac{I_{i,t}}{(\delta + g_{i,t})} \quad \text{(A2.1.7)}$$

$$\mu_{i,t} = r_{i,t}p_{i,t-1} + d_{i,t}p_{i,t} - \left(p_{i,t} - p_{i,t-1}\right) \quad \text{(A2.1.8)}$$

$$\ln\left[\frac{K_t}{K_{t.1}}\right] = \sum_i \bar{v}_i \ln[S_{i,t}/S_{i,t-1}] \text{(A2.1.9)}$$

$$v_{i,t} = \frac{\mu_{i,t}S_{i,t}}{\sum_i \mu_{i,t}S_{i,t}} \quad \text{(A2.1.10)}$$

A2.2 United Kingdom

Capital stocks are computed using the perpetual inventory method (PIM): $S_{ij,t}$ represents the stock of i-th asset ($i = 1, \dots, N$) in j-th industry ($j = 1, \dots, M$) at time t; the depreciation rate d_i is assumed geometric, constant and equal for all industries; gross investment is represented by $I_{ij,t}$. Capital stocks then grow over time in accordance with

$$S_{ij,t} = I_{ij,t} + (1 - d_i)S_{ij,t-1} \quad \text{(A2.2.1)}$$

Starting stocks in the beginning of the period t are based on the dataset underlying Wallis (2009), which is fully consistent with historic ONS capital stock data. Oulton & Wallis (2015) consider 9 asset categories: structures, machinery, vehicles, computer, own-account software, purchased software, mineral exploration, artistic originals, and R&D. Investment data from 1997 on is taken from regular ONS business investment releases[16], and supplemented by the authors with ad hoc releases on software, artistic originals and mineral exploration[17]. All data pre-1997 is taken from the 2003 release of investment data underlying previous ONS capital stock estimates[18]. This data is spliced with the latest estimates from 1997 onwards. Depreciation rates are the same as the ones used historically for official capital estimates (see Oulton & Wallis, 2015).

Aggregate capital stock in the j-th industry is calculated as a Törnqvist index:

$$\ln\left(\frac{S_{j,t}}{S_{j,t-1}}\right) = \sum_{i=1}^{i=N} \overline{w}_{ij,t}^{S} \ln\left(\frac{S_{ij,t}}{S_{ij,t-1}}\right) \qquad \text{(A2.2.2)}$$

Where the weights are $\overline{w}_{ij,t}^{S} = 0.5(w_{ij,t}^{S} + w_{ij,t-1}^{S})$ and $w_{ij,t}^{S}$ is

$$w_{ij,t}^{S} = \frac{p_{ij,t}^{S} S_{ij,t}}{\sum_{i=1}^{i=N} p_{ij,t}^{S} S_{ij,t}} \qquad \text{(A2.2.3)}$$

With $p_{ij,t}^{S}$ being the price of a unit of capital of the i-th type (asset price).

Capital services delivered by any asset during period t are assumed proportional to the stock of that asset at the end of the period $t-1$ with the constant of proportionality normalized to unity: $K_{ij,t} = S_{ij,t-1}, i = 1, \dots, N; j = 1, \dots, M$. Aggregate capital services in the j-th industry are also calculated as a Törnqvist index, where the weights are the shares in industry profit attributable to each asset:

$$\ln\left(\frac{K_{j,t}}{K_{j,t-1}}\right) = \sum_{i=1}^{i=N} \overline{w}_{ij,t}^{K} \ln\left(\frac{K_{ij,t}}{K_{ij,t-1}}\right) \qquad \text{(A2.2.4)}$$

With $\overline{w}_{ij,t}^{K} = 0.5(w_{ij,t}^{K} + w_{ij,t-1}^{K})$ and $w_{ij,t}^{K}$ is

$$w_{ij,t}^{K} = \frac{p_{ij,t}^{K} K_{ij,t}}{\sum_{i=1}^{i=N} p_{ij,t}^{K} K_{ij,t}} \qquad \text{(A2.2.5)}$$

By definition, the value of capital services equals profit or gross operating surplus (GOS):

$$\sum_{i=1}^{i=N} p_{ij,t}^{K} K_{ij,t} = GOS_{j,t} \qquad \text{A(2.2.6)}$$

Here, $p_{ij,t}^{K}$ are the rental prices (user costs) of capital services, given by the Hall & Jorgenson (1967) formula:

$$p_{ij,t}^{K} = T_{ij,t}[r_{j,t} + d_i(1 + \pi_{ij,t}) - \pi_{ij,t}]p_{ij,t-1}^{S} \qquad \text{(A2.2.7)}$$

16 http://www.ons.gov.uk/ons/rel/bus-invest/business-investment/index.html

17 http://www.ons.gov.uk/ons/about-ons/business-transparency/freedom-of-information/what-can-i-request/published-ad-hoc-data/index.html

18 http://www.ons.gov.uk/ons/publications/re-reference-tables.html?edition=tcm%3A77-31299

Where $T_{ij,t}$ is a tax adjustment factor, taken from Wallis (2012) and varying by asset but not by industry. The nominal rate of return $r_{j,t}$ is calculated under the endogenous (ex-post) approach, and assumed the same for all assets. $\pi_{ij,t}$ is the rate of growth of the i-th asset price:

$$\pi_{ij,t} = (p_{ij,t}^{S} - p_{ij,t-1}^{S})/p_{ij,t-1}^{S} \qquad \text{(A2.2.8)}$$

A3 – useful work methodology

The most extensive useful exergy accounting study focusing on the Portuguese economy has been conducted by Serrenho et al. (2015). This study covers a 154-year period from 1856 to 2009, and focuses analysis on final-to-useful processes. The step-by-step methodology applied for each year and energy carrier is adopted from the approach by Warr et al. (2010).

The step-by-step methodology is as follows:

1) Conversion of existing final energy data[19] to final exergy values;
2) Allocation of final exergy consumption of each final use sector to useful exergy categories;
3) Estimation of second-law efficiencies for each final-to-useful transformation;
4) Calculation of aggregate useful exergy values by summing total values obtained for each useful exergy category.

The authors refine the useful exergy accounting methodology by taking into account differences in final-to-useful electricity uses. Besides the typical energy carriers (coal & coal products, oil & oil products, natural gas, combustible renewables, electricity & CHP heat), the authors also take into account energy/exergy inputs that go beyond conventional energy accounting statistics: food for humans, feed for working animals, and non-conventional sources[20]. The dataset used is compiled from different sources, for final energy consumption: IEA energy balances (typical energy carriers after 1960); Henriques (2011) (typical energy carriers and food/feed before 1960, as well as corrections for IEA combustible renewables data prior to 1990 and non-conventional energy carriers for the entire period); FAO (food/feed after 1960).

Serrenho et al. (2015) consider the following categories for energy end-use: heat (high, medium and low temperature); mechanical drive; light; electricity[21]; and muscle work.

[19] The authors define final energy consumption as the total effective consumption, i.e. standard final energy consumption as commonly defined in official energy statistics plus energy sector own energy uses.

[20] e.g. wind and water streams for mechanical drive uses in boats, mills and wells.

[21] Electricity is treated separately, since it can be used either for heating, lighting, mechanical drive, or other electric uses, depending also on the sector where it is used.

For each year (t) and each combination of energy carrier (i), economic sector (j), and energy end-use category (k), useful exergy is calculated as follows:

$$X_{U\,t,ijk} = \epsilon_{t,k} \varphi_i E_{F\,t,ijk} \qquad \text{(A3.1)}$$

The process requires a mapping for energy-uses, estimation of thermodynamics 2nd law efficiencies for each end-use category ($\epsilon_{t,k}$), and the definition of an exergy factor[22] for each energy carrier (φ_i). The mapping depends on the level of disaggregation of the energy data for final energy consumption ($E_{F\,t,ijk}$). For details on the estimation of 2nd law efficiencies and exergy factors in this study, consult Serrenho et al. (2015).

A more recent useful exergy accounting study for Portugal, focusing only on the years between 1960 and 2009, has been conducted by Palma (2014). This study follows the same methodology as Serrenho et al. (2015), focusing on the conversion from final to useful energy/exergy. This study introduces some important changes to the otherwise similar methodology adopted from Serrenho et al. (2015): 1) cooling is introduced as a new category for energy end-uses; 2) heating efficiencies are refined to also depend on the energy carrier; 3) a more detailed disaggregation of electricity end-uses per sector. Other, smaller changes include: a) Palma's (2014) study does not take into account food & feed energy carriers; b) electricity and CHP heat energy carriers are treated separately; c) mechanical drive end-use category is divided between transport and stationary mechanical drive.

Although being used in other similar studies (Brockway et al. 2014), cooling has not been accounted for as a separate energy end-use category, instead being included as stationary mechanical drive or other electric uses. Cooling uses arise from electricity, and are related to space cooling uses in all sectors, through air conditioners, and refrigeration uses mainly in the industrial, residential and services sectors. As a service provided, it has a significant share in final exergy estimates.

Likewise, the disaggregation of heating process efficiencies by energy carrier – as opposed to previous studies, in which efficiencies for heat processes were assumed constant among energy carriers – results in higher efficiencies for natural gas than for coal and combustible renewables.

Finally, while Serrenho et al. (2015) allocate electricity end-uses for the electricity energy carrier only between the industrial sector and all other sectors, Palma (2014) includes shares of utilization for each of the sectors (industrial, transport, residential, services and miscellaneous), producing a more accurate result of the distribution of useful exergy in each sector.

[22] Defined as the ratio of exergy to energy.

Appendix B – Solow Residual derivations

B1 –Cobb-Douglas

Estimating the Solow residual. Assuming that parameters θ, α and β in Equation 2 are known from parameter estimation, we can estimate the value of λ as follows. First, assume constant returns to scale such that $\alpha + \beta + \gamma = 1$, and calculate $\gamma = 1 - \alpha - \beta$. Then, take the natural logarithm (ln) of Equation 2 to obtain

$$lny = ln\theta + lnA + \alpha lnk + \beta lnl + (1 - \alpha - \beta)lne. \qquad \text{(B1.1)}$$

By taking the derivative of Equation B1.1 with respect to time (t) and noting that model parameters θ, α, and β are constant with respect to time, we obtain

$$\frac{1}{y}\frac{dy}{dt} = \frac{1}{A}\frac{dA}{dt} + \alpha\frac{1}{k}\frac{dk}{dt} + \beta\frac{1}{l}\frac{dl}{dt} + (1 - \alpha - \beta)\frac{1}{e}\frac{de}{dt} \qquad \text{(B1.2)}$$

Solving for the total factor productivity term gives

$$\frac{1}{A}\frac{dA}{dt} = \frac{1}{y}\frac{dy}{dt} - \left[\alpha\frac{1}{k}\frac{dk}{dt} + \beta\frac{1}{l}\frac{dl}{dt} + (1 - \alpha - \beta)\frac{1}{e}\frac{de}{dt}\right] \qquad \text{(B1.3)}$$

Recognising that $A \equiv \exp[\lambda(t - t_0)]$ gives

$$\frac{1}{A}\frac{dA}{dt} = \frac{1}{\exp[\lambda(t-t_0)]}\lambda\exp[\lambda(t - t_0)] = \lambda \qquad \text{(B1.4)}$$

which can be substituted into Equation B1.3 to find

$$\lambda = \frac{1}{y}\frac{dy}{dt} - \left[\alpha\frac{1}{k}\frac{dk}{dt} + \beta\frac{1}{l}\frac{dl}{dt} + (1 - \alpha - \beta)\frac{1}{e}\frac{de}{dt}\right] \qquad \text{(B1.5)}$$

Equation B1.5 applies for any instant in time.

If we approximate derivatives in Equation B1.5 with forward differences between time i and j, we find

$$\lambda_{i,j} = \frac{1}{y_i}\frac{y_j - y_i}{t_j - t_i} - \left[\alpha\frac{1}{k_i}\frac{k_j - k_i}{t_j - t_i} + \beta\frac{1}{l_i}\frac{l_j - l_i}{t_j - t_i} + (1 - \alpha - \beta)\frac{1}{e_i}\frac{e_j - e_i}{t_j - t_i}\right] \qquad \text{(B1.6)}$$

where $\lambda_{i,j}$ approximates the true, instantaneous value of λ from Equation B1.5 between times i and j.

Interestingly,

$$\lambda_{1,n} \neq \sum_{i=2}^{n}\lambda_{i,i-1} \qquad \text{(B1.7)}$$

Rather,

$$\lambda_{1,n} = \frac{1}{y_1}\frac{y_n - y_1}{t_n - t_1} - \left[\alpha \frac{1}{k_1}\frac{k_n - k_1}{t_n - t_1} + \beta \frac{1}{l_1}\frac{l_n - l_1}{t_n - t_1} + (1 - \alpha - \beta)\frac{1}{e_1}\frac{e_n - e_1}{t_n - t_1}\right] \quad \text{(B1.8)}$$

which will become increasingly inaccurate over large time spans, because y_1, k_1, l_1 and e_1 will be less representative of the average value of y, k, l and e in the time span, respectively. This suggests that an averaging approach such as

$$\lambda_{1,n} = \frac{\sum_{i=2}^{n} \lambda_{i,i-1}}{n-1} \qquad \text{(B1.9)}$$

is a better approximation of Equation B1.8, which is, in itself, an approximation of the true value of λ given by Equation B1.5.

Fraction of growth explained by the Solow residual. The fraction (f) of GDP growth explained by the Solow residual can be given as

$$f \equiv \frac{\lambda}{\frac{1}{y}\frac{dy}{dt}} = 1 - \frac{\alpha\frac{1}{k}\frac{dk}{dt} + \beta\frac{1}{l}\frac{dl}{dt} + (1-\alpha-\beta)\frac{1}{e}\frac{de}{dt}}{\frac{1}{y}\frac{dy}{dt}} \quad \text{(B1.10)}$$

Using forward differences to estimate f yields

$$f_{i,j} \equiv \frac{\lambda_{i,j}}{\frac{1}{y_i}\frac{y_j - y_i}{t_j - t_i}} = 1 - \frac{\alpha\frac{1}{k_i}\frac{k_j - k_i}{t_j - t_i} + \beta\frac{1}{l_i}\frac{l_j - l_i}{t_j - t_i} + (1-\alpha-\beta)\frac{1}{e_i}\frac{e_j - e_i}{t_j - t_i}}{\frac{1}{y_i}\frac{y_j - y_i}{t_j - t_i}} \quad \text{(B1.11)}$$

Cancelling $t_j - t_i$ terms and simplifying gives

$$f_{i,j} = 1 - \frac{\alpha\left(\frac{k_j}{k_i} - 1\right) + \beta\left(\frac{l_j}{l_i} - 1\right) + (1-\alpha-\beta)\left(\frac{e_j}{e_i} - 1\right)}{\left(\frac{y_j}{y_i} - 1\right)} \qquad \text{(B1.12)}$$

which is an estimate of the instantaneous value of f given by Equation B1.10.

B2 –CES function