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Book of Abstracts

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British Logic Colloquium

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20. Kai Ino (University of Manchester)
21. Mark Kamsma (University of East Anglia)
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Mini-course

NIP fields and valued fields

Sylvy Anscombe
University of Central Lancashire

This minicourse is about NIP fields and the problem of finding an algebraic characterization. Though it may be considered just one of a family of similar hard problems (e.g. stable fields, simple fields), we will see that this topic has been gathering momentum in recent years. I aim to cover the main developments without assuming any specialist knowledge, for example from valuation theory.

Outline of topics:

- Definitions and discussion of the place of fields in the stability-theoretic world
- Examples of NIP fields
- Valued fields, henselianity
- the conjecture
- ACVF, Ax–Kochen/Ershov, and examples
- Examples of NIP valued fields
- The theorem of Kaplan–Scanlon–Wagner
- dp-minimality and Johnson's work
- strongly dependent fields
- conjectural classification
- Halevi–Hasson–Jahnke, topological henselianity.

Wednesday 8th

Small groups of finite Morley rank with a tight automorphism

Ulla Karhumäki

University of Manchester

We explain in details "the Hrushovski Programme for infinite simple groups of finite Morley rank". The main aim of this programme is to prove that the well-known Cherlin-Zilber Algebraicity Conjecture is equivalent to another conjecture called the Principal Conjecture; *Let G be an infinite simple group of finite Morley rank with a generic automorphism α. Then, the fixed points subgroup $C_G(\alpha)$ is pseudofinite.* We prove a partial result towards the expectation that these conjectures are equivalent. Namely, we prove that an infinite simple group of finite Morley rank G with a Prüfer 2-rank 1 admitting a tight automorphism α is isomorphic to $\mathrm{PSL}_2(K)$ over an algebraically closed field K of characteristic different from 2. This is a joint work with Pınar Uğurlu.

Geometries in sharply 2-transitive groups of finite Morley rank

Tim Clausen

Westfälische Wilhelms-Universität Münster

A group G acting on X is sharply 2-transitive if it acts regular on pairs of distinct points. The standard example of such a group is $\mathrm{AGL}_1(K)$ where K is a field. Conjecturally, all infinite sharply 2-transitive groups of finite Morley rank should be standard. If G is an infinite sharply 2-transitive group of finite Morley rank such that its characteristic is not equal to two, then G admits a point-line geometry on the set of involutions. We study this geometry and its subgeometries to improve known rank inequalities.

Syndetic sets in definable groups

Margarita Otero
Universidad Autónoma de Madrid

By definable group we mean a definably connected group G definable in an o-minimal structure. A subset X of G is syndetic if there is a finite subset F of G such that $FX = G$ (X is also called generic in the definable context). If G is definably compact, we have a nice description of G via abelian subgroups: $G = T^G$, where T is a maximal abelian definably connected subgroup of G.

In the general case we cannot hope for so much.

Conjecture. *If G is a definably connected definable group, there is a maximal nilpotent subgroup H of G such that H^G is syndetic in G.*

I will talk about this conjecture.

Bibliography

[1] E. Baro, A. Berarducci, and M. Otero. Cartan subgroups and regular points of o-minimal groups. *J. London Math. Soc.*, 100(2):361–382, 2019.

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A nondefinability result for Weierstrass $\wp$ functions

Raymond McCulloch
University of Manchester

In 1997 Bianconi combined some ideas from Wilkie's proof of model completeness of the real exponential field with a result due to Ax, to show that no restriction of the sine function to a finite real interval is definable in the real exponential field. Equivalently no restriction of complex exp to a disc is definable using real exp. In this talk I shall discuss a similar problem for some related transcendental functions, the Weierstrass $\wp$ functions. I will begin by recording an observation of Macintyre before stating the converse to Macintyre's result and making some remarks on the proof.

Integer-valued definable functions in $\mathbb{R}_{\text{an,exp}}$

Shi Qiu
University of Manchester

The interaction between model theory and diophantine geometry began with a new proof, by Pila and Zannier, of the Manin-Mumford conjecture. And this led to a breakthrough with Pila's proof of the Andre-Oort conjecture for products of modular curves. In a related direction, Jones, Thomas and Wilkie[2012] applied improvements of the Pila-Wilkie theorem for certain curves to prove results on integer-valued functions, that is, functions f such that $f(n)$ is an integer for integer points in the domain of f definable in the real exponential field. This gives a version of a 100 year old theorem due to Polya, but with complex functions replaced by real functions.

Theorem 1 (Polya)**.** *If $f : \mathbb{C} \to \mathbb{C}$ is entire with $f(\mathbb{N}) \in \mathbb{Z}$ and $|f(z)| \leq dC^{|z|}$ with real d and $C < 2$, then f is a polynomial.*

More recently, Wilkie[2016] proved an almost exact analogue of Polya's theorem in $\mathbb{R}_{\text{an,exp}}$. This talk will show how to combine Wilkie's ideas with techniques from transcendental number theory and o-minimality in order to establish Polya-type theorems in which the function is definable in o-minimal expansion of real ordered field and only assumed to be integer valued on a certain sequence of natural numbers.

Thursday 9th

Transfer principles and applications

Konstantinos Kartas
University of Oxford

In this talk we will focus on methods that can be used to transfer first-order information (e.g. decidability) from equicharacteristic p valued fields to mixed characteristic valued fields. Given a mixed characteristic valued field K, we can construct a characteristic p analogue of K. This field, at least when high ramification is present, will share many number-theoretic properties in common with K. It was an approach initiated by Krasner and developed further by Kazhdan-Deligne to deduce information about K using its characteristic p approximation. We will discuss how this philosophy is also applicable in a model-theoretic context.

Defining henselian valuations in NIP fields, explicitly and conjecturally

Blaise Boissonneau
Westfälische Wilhelms-Universität Münster

A famous conjecture attributed to Shelah ties the understanding of NIP pure fields with henselianity by stating that a non-trivial NIP field should admit a non-trivial henselian valuation. But since the NIPity hypothesis concerned only a pure field, where does this valuation come from? It turns out that it was there all along, that is, if Shelah conjecture holds, then a non-trivial NIP field admits a non-trivial henselian valuation explicitly definable in the language of rings.

Normal subgroups of automorphism groups of countable structures

David Evans

Imperial College London

Following earlier work of Lascar and of Macpherson and Tent, Tent and Ziegler introduced the general machinery of stationary independence relations (SIR) and used this to prove that the automorphism groups of certain countable structures are simple, or simple modulo a subgroup of 'bounded' automorphisms. I will describe some generalisations and applications of these results. In particular, I will discuss recent work of Yibei Li which removes the assumption of symmetry in the definition of a SIR.

Extending partial automorphisms of structures

Jan Hubička and Matěj Konečný

Charles University, Prague

The extension property for partial automorphisms (EPPA), also called the Hrushovski property, is a property of classes of finite structures stating that for every A there is B (its EPPA-witness) containing A as a substructure such that every isomorphism of substructures of A extends to an automorphism of B. Every class with EPPA is an amalgamation class, in fact, EPPA is equivalent to the fact that the automorphism group of the Fraïssé limit of the class can be written as the closure of an increasing chain of proper compact subgroups. In particular, EPPA is a key ingredient in proving ample generics, the small index property etc.

In this talk, we present a general construction for EPPA-witnesses in languages containing relations and unary functions, such that one has strong control over their local structure. In particular, this is a common strengthening of theorems of Herwig and Lascar, Hodkinson and Otto and Siniora and Solecki. Joint work with Jaroslav Nešetřil.

On piecewise hyperdefinable groups and applications

Arturo Rodríguez Fanlo
University of Oxford

Piecewise hyperdefinable sets are natural generalizations of interpretable sets. A standard example is the quotient of a subgroup generated by a definable set over an $\bigwedge$-definable (infinitely-definable, type-definable) normal subgroup. They have natural logic topologies, which extend the usual Stone topology, given by taking as closed sets the piecewise $\bigwedge$-definable subsets.

Firstly, we will discuss the general structure of piecewise hyperdefinable sets and their main topological properties. After that, we will focus on the case of piecewise hyperdefinable groups, where we find associated Lie groups, which we call Lie cores. Our main results, based on previous results of *Stable group theory and approximate subgroups* (Hrushovski, 2011), show their uniqueness and sufficient and necessary conditions for their existence.

Applying then a generalized version of the stabilizer theorem for piecewise hyperdefinable groups, we can extend the results of *Stable group theory and approximate subgroups*, getting also possible applications to metric approximate subgroups.

Friday 10th

O-minimality and special point problems

Guy Fowler
University of Oxford

Pila and Tsimerman (2017) proved a result on the finiteness of multiplicatively-dependent n-tuples of singular moduli (i.e. j-invariants of CM elliptic curves). The proof is by o-minimality, in particular, the counting theorem of Pila–Wilkie. In common with other results proved by this method, it is ineffective. This talk will cover some extensions of Pila–Tsimerman's result, and in particular an effective version of it for $n \leq 3$.

A category-theoretic version of the Kim-Pillay theorem

Mark Kamsma
University of East Anglia

In Shelah's classification of first-order theories using combinatorial properties, the notion of a stable theory is the most well-known. Stable theories are very well-behaved. The notion of a simple theory is a generalisation of this. The theory of the random graph is the prototypical example of a simple theory. In particular, one can develop the concept of forking independence in these theories. This is a generalisation of linear independence in vector spaces, for example. The Kim-Pillay theorem gives us a way to characterise simple theories based on the existence of a certain independence relation. It states that if an independence relation of a certain form exists, the theory is simple and that this independence relation coincides with forking independence. For example, for the random graph this can be applied to the independence relation that says that sets A and B are independent over C if $A \cap B \subseteq C$. We will recall all this in more detail at the start of the talk. After that, we will set up a category-theoretic framework where we can make sense of certain model-theoretic tools and definitions. Taking inspiration from work of Lieberman, Vasey and Rosický, we can define what an independence relation is in this framework. This then allows us to formulate a category-theoretic version of the Kim-Pillay theorem. We will finish by looking at a few examples of where this framework applies, including positive logic and continuous logic. In particular, we will recover the original Kim-Pillay theorem as one of these applications.

Some Ramsey theory and topological dynamics for first order theories

Krzysztof Krupiński

Uniwersytet Wrocławski

A fundamental question for a given complete first order theory T is whether Kim-Pillay strong types coincide with Shelah strong types; this is equivalent to the KP-Galois group of T being profinite. For example, a well-known open problem is whether it holds for simple theories. On the other hand, in my paper with A. Pillay and T. Rzepecki, we introduced the so-called Ellis group of a theory which captures more information about the theory than any of its Galois groups. One can deduce from our paper that if this Ellis group (or its canonical Hausdorff quotient) is profinite, then so is the KP-Galois group of T. So in the present joint work with J. Lee and S. Moconja, we try to understand when the Ellis group of a given theory is profinite. We have found various criteria for it which in particular include new connections with structural Ramsey theory. We have introduced natural notions of [externally] definable Ramsey properties and degrees for a first order theory, extending classical embedding Ramsey properties considered in Kechris-Pestov-Todorčević theory. Among the main results are theorems providing dynamical characterizations (e.g. via [extreme] amenability of the theory in question, or via some properties of the Ellis semigroups of some flows associated with the theory) of our Ramsey properties, some of which imply that the Ellis group of the theory in question is profinite.

Definable connectedness of randomizations of groups

Jorge Muñoz Carvajal

Université Claude Bernard Lyon 1

The randomization of a structure M is a continuous structure built by taking random variables whose values belong to M. Many desirable model-theoretic properties such as ω-categoricity, ω-stability, stability and NIP, are preserved by the randomization. In this talk, we show that the randomization of a stable group or a definably amenable NIP group are definably connected. This is a joint work with A. Berenstein.

Independence relations in ACFG

Christian d'Elbée
Hebrew University of Jerusalem

The model-theory of a vector space entirely follows from the presence of a notion of independence: the linear independence. Basically, every type is determined by the one of an independent set (a basis) which is unique and has a unique generic extension. This interaction between model theory and a notion of independence manifest itself in classical subjects of classification theory: stability and simplicity. Both stability and simplicity are characterized by the presence of a "well behaved" notion of independence, based on the notion of Shelah's forking. In the more general NSOP_1 theories, there is also an equivalent characterization in term of a notion of independence, based on the so-called Kim-forking. ACFG is an NSOP_1 non-simple theory, it is the model-companion of an algebraically closed field of fixed positive characteristic with a predicate for an additive subgroup. Every model of this theory is equipped with various independence relations more or less well behaved. In ACFG, all the nice properties that are concentrated in the forking independence relation in stable theories appears but in various independence relations, as if the "stable" independence splits into different independence relations in an unstable (or unsimple) context. In this talk, we give a survey of those various independence relations in ACFG, as well as interactions between them.

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