Families of unimodal distributions on the circle

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I will start this talk on the real line, briefly describing some families of unimodal distributions with three or four parameters, controlling location, scale, skewness and perhaps some aspect(s) of tailweight. Can this technology be transferred to the case of distributions on the circle? The answer is a qualified yes, unimodality apparently being somewhat harder to retain in this case (except by the unattractive technique of wrapping).

In particular, I will describe three families of four-parameter unimodal circular distributions which arise, both being particular instances of the relatively obscure linear notion of "transformation of scale" distributions. The first two will be "direct" and, my particular speciality, "inverse" Batschelet distributions (Jones & Pewsey, 2012, Biometrics). The third, on which most time will be taken, appears to prove even better (work in progress; details not completely finalised!) and is the brainchild of my excellent collaborator Shogo Kato. As well as unimodality, this family has numerous attractive properties. It has: a simple characteristic function and can be parameterised directly in terms of its first four trigonometric moments; also, tractable density and distribution functions; a very wide range of skewness and "kurtosis"; nice sub-models including the wrapped Cauchy and cardioid distributions; straightforward parameter estimation by both method of moments (suitable for smaller samples and moderate parameter values) and maximum likelihood; closure under convolution.

I will remain obsessed with unimodality throughout the talk, taking the view that bi- and multi-modality can be interpretably modelled by mixtures, or in the circular case perhaps by "multiplicative mixtures", of which the distributions of interest may form components.