British Postgraduate Model Theory Conference

Book of Abstracts

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Contents

Invited Talks

Model theory of infinite Steiner triple systems

Silvia Barbina

Univeristy of Camerino

Finite Steiner Triple Systems (STs) are well-known combinatorial objects for which the literature is extensive. An STS is a set S together with a collection B of subsets of S of size 3 such that any two elements of S belong to exactly one element of B.

Recently, the infinite counterparts of these structures have proven interesting from a model-theoretic viewpoint. We will survey recent results in the area, in particular regarding the countable universal homogeneous STS and free constructions of STSs.

Definably complete exponential fields

Alessandro Berarducci

Università di Pisa

Let $\mathbb{R}_{\exp}$ be the real field with the exponential function. This structure satisfies the axioms of ordered fields, the axiom scheme of definable completeness (a first-order analogue of completeness), and an axiom expressing the fact that exp is a differentiable function satisfying the differential equation $\mathsf{exp}'(x) = \mathsf{exp}(x)$ and $\mathsf{exp}(0) = 1$. Assuming Schanuel's conjecture and building on the work of Wilkie, we shall consider the problem of whether this is a complete axiomatisation (work in progress with Marcello Mamino). Related conjectures can be raised for the exponential function restricted to a bounded interval.

Measures on perfect PAC fields

Zoé Chatzidakis

CNRS (IMJ-PRG), Université Paris-Cité

A conjecture, now disproved by Chernikov, Hrushovski, Kruckman, Krupinski, Pillay and Ramsey, asked whether any group with a simple theory is definably amenable.

It is well known that the counting measure on finite fields gives rise to a non-standard counting measure on pseudo-finite fields (the infinite models of the theory of finite fields). It was unknown whether other PAC fields possessed a reasonable measure, and in this talk, we will show that some of them do, although the measure we define does not have all the nice properties of a counting measure when the field is not pseudo-finite. This result can be used to show that if G is a group definable in an e-free or omega-free perfect PAC field, then G is definably amenable.

We hope to extend our results to a wider class of perfect PAC fields. I will also discuss the case of groups definable in certain bounded PRC fields.

This is work in progress, joint with Nick Ramsey (Notre Dame).

Definability patterns and structural Ramsey theory (Mini-course)

David Evans

Imperial College London

I will discuss some aspects of Hrushovski's work on definability patterns (arXiv:1911.01129) and its connections to structural Ramsey theory and the Kechris-Pestov-Todorcevic correspondence. As well as Hrushovski's paper, the lectures will be based on some notes of Pierre Simon and talks given in a joint London-Prague seminar during 2022.

Suppose L is a first-order language and T is a universal L-theory which is *irreducible*, meaning that its class of models has

the Joint Embedding Property. If V is a sort (or collection of sorts) associated to T, then Hrushovski shows that there is a canonical *minimal Ramsey expansion* T_V^{Ram}. We will explain what this means, talk about the connections to structural Ramsey theory and KPT and discuss the proof. The construction of T_V^{Ram} depends on Hrushovski's construction of the *core* of T for free pattern types at V, which is a special case of a more general object. We will focus specifically on explaining what this means in the Ramsey context.

Contributed Talks

Many flavors of CHIPS

Blaise Boissonneau

KU Leuven

In this talk, we will study transfer theorems for valued fields; specifically, we will expose a method first developed by Chernikov and Hils in 2014 for NTP_2 transfer, before being adapted for NIP transfer by Jahnke and Simon in 2018. This method gives two sufficient conditions for transfer to happen: the value group and residue field are stably embedded, and types of immediate extensions are well-behaved. We will give a heuristic for why these conditions guarantee transfer, present a NIP_n transfer theorem, and give a complete classification of henselian valued fields having NIP_n transfer.

A characterization of ω-pseudo complete valued fields elementarily equivalent to a finite extension of $\mathbb{Q}_p$

Anna De Mase

Università degli Studi della Campania "L. Vanvitelli"

In 1965 Ax and Kochen gave a characterisation of ω-pseudo complete models of the theory of the field of p-adic numbers through an Hahn-like construction over $\mathbb{Q}_p$. We generalise this construction to any mixed characteristic henselian valued field with finite ramification and obtain an analogous characterisation for ω-pseudo complete valued fields elementarily equivalent to a fixed finite extensions of $\mathbb{Q}_p$.

Dividing lines in positive unstable theories

Anna Dmitrieva

University of East Anglia

Thursday 16:30-17:00

The classification theory, started by Shelah around 1970, has been one of the prominent themes in model theory. The results, accumulated in the famous "Map of the Universe", divide theories depending on how complicated the behaviour of their definable sets is. In a joint work with Francesco Gallinaro and Mark Kamsma we extend some of these classical results to the setting of positive logic, concentrating on the unstable case. We also show that the two definitions of stable and simple theories are still equivalent in positive logic and obtain a weaker analogue of stable being the same as NIP and NSOP, replacing NSOP with NSOP_1.

Monadic NIP as the limit to tractable computation

Ioannis Eleftheriadis

University of Cambridge

Wednesday 12:30-13:00

It is conjectured that the hereditary classes of structures that admit fixed-parameter tractable model-checking are precisely those that are (monadically) NIP. We provide a positive answer to this conjecture in the case of monotone classes of arbitrary relational structures. This is established by showing that (monadic) NIP coincides with having nowhere dense Gaifman class for monotone classes.

Joint work with Sam Braunfeld, Anuj Dawar, and Aris Papadopoulos.

Forking and extension bases in Henselian valued fields

Akash Hossain

Université Paris-Saclay

The study of the model-theoretic properties of Henselian valued fields (of residue characteristic zero) has been a rich subject which led to applications in number theory, algebraic geometry, and more recently analytic geometry. One of the key modern model-theoretic properties of these structures that is not yet well-understood is the behaviour of forking. While understanding this notion in full generality proves to be a hard subject, in this talk we will emphasise on extension bases, defined as those parameter sets over which every type is a non-forking extension of itself. Various results in the literature show that many neostability properties hold over those sets. In this talk, after a brief introduction to extension bases and valued fields, we will give an overview of what is already known about forking in those structures: the results about separated extensions established by Haskell-Hrushovski-Macpherson, and later extended by Vicaria and Ealy-Haskell-Simon. We will finish by stating our results giving sufficient conditions for a parameter set to be an extension base. These conditions do not require said set to be a maximal immediate valued field, hence they apply to extensions that are not separated, unlike the previous results of the literature. We show in particular that every set in the theory of any ultraproduct of the p-adic fields (of residue characteristic zero) is an extension base, which implies that forking equals dividing.

Differentially closed fields in positive characteristic

Friday
11:30-12:00

Kai Ino

The University of Manchester

In the 1970s, Carol Wood studied the model theory of differentially closed fields in positive characteristic, and she showed the existence of prime model extension over each differentially perfect field. We have studied a new class of differential fields called separably differentially closed fields by generalizing her work and showing the existence of prime model extension over each differential field. I will explain this by comparing her work and the case of characteristic zero.

Invariant Keisler measures in NIP omega-categorical structures

Friday
12:00-12:30

Paolo Marimon

Imperial College London

In joint work with Samuel Braunfeld we give a full classification of invariant Keisler measures in NIP omega-categorical structures. This completes the work of Ensley, who gave a classification under some additional assumptions.

Automorphisms of ordered abelian groups, the Amalgamation Property, and dependent positive theories

Rosario Mennuni

Università di Pisa

Wednesday
12:00-12:30

I will talk about recent joint work with Jan Dobrowolski, where we study ordered abelian groups with an automorphism. We prove that their embeddings have the Amalgamation Property. Since the existentially closed ones do not form an elementary class, we study them in the framework of positive logic, develop a notion of NIP for this context, and show that the aforementioned class is NIP this is generalised sense.

Fuzzy Semantics for Expressively Rich Languages

Theodor Nenu

University of Bristol

Wednesday
16:30-17:00

The phenomena of vagueness and self-reference show that a handful of intuitive truth-theoretic principles cannot simultaneously be upheld for all sentences of any language capable of expressing basic syntactic facts. Alfred Tarski's (1935) seminal article presented a couple of metamathematical challenges towards providing classical models for formalised languages containing type-free truth-predicates. Some of the usual obstacles can be elegantly bypassed in a fuzzy framework: we will show how to provide fuzzy models of the first-order language of arithmetic enriched with a monadic disquotational truth-predicate—but also explain why we cannot obtain fuzzy models anymore if we augment that language with further truth-symbols for precise degreed truth. Towards the end of the paper, we will investigate the prospects of offering fuzzy models for a couple of expressively rich languages where Kripke's (1975) famous transfinite-inductive construction cannot be deployed because of monotonicity failure (e.g. modal

languages including primitive non-truth-functional counterfactual connectives).

Thursday
17:30-18:00

Henselian Fields with Generic Derivations

Gabriel Ng

University of Manchester

In this talk, we will discuss the class of valued-differential fields which are henselian as pure valued fields and whose derivation satisfies a certain genericity condition, which we call differentially henselian. In particular, we will discuss the connections between differentially henselian fields and differentially large fields. We shall exhibit examples, alternative characterisations, and an Ax-Kochen/Ershov type theorem in certain classes of these fields.

Friday
17:00-17:30

Absolutely Pure Filtered Modules

Soinbhe Nic Dhonncha

University of Manchester

Fix a ring R. The absolutely pure objects of the category of filtered modules over R, considered as a subcategory of $\mathrm{Mod}RA_\infty^\infty$, are trivial; they correspond exactly to the absolutely pure objects of $\mathrm{Mod}R$. The reason for this is tied to the quasi-abelian nature of the category. We describe a much larger class of objects which we will call "strictly pure" that are absolutely pure with respect to a natural class of their superstructures.

Elementary classes of real closed SV-rings.

Ricardo Jesus Palomino Piepenborn

The University of Manchester

Real closed SV-rings are rings of functions from a non-empty set to a real closed field R which are closed under all continuous 0-definable functions from R^n to R and whose quotient modulo prime ideals is a valuation ring. The Zariski spectrum of real closed SV-rings gives a rough measure of their complexity, and using suitable enrichments of the language of rings one can capture in a first-order way the structure of these spectra; in particular, the class of real closed SV-rings whose spectrum has bounded rank is elementary in the language of rings. In this talk I'll motivate the model-theoretic study of this class of rings, introduce the main definitions and examples, and provide some first illustrative results; time permitting, I'll explain some connections between real closed SV-rings with finite spectrum and tame pairs of real closed fields.

Maximal Stable Quotients of Invariant Types In NIP Theories

Adrián Portillo Fernández

University of Wrocław

For a NIP theory T, a sufficiently saturated model $\mathfrak{C}$ of T, and an invariant (over some small subset of $\mathfrak{C}$) global type p, we prove that there exists a finest relatively type-definable over a small set of parameters from $\mathfrak{C}$ equivalence relation on the set of realisations of p which has stable quotient. This is a counterpart for equivalence relations of the main result of [1] on the existence of maximal stable quotients of type-definable groups in NIP theories. Our proof adapts the ideas of the proof of this result, working with relatively type-definable subsets of the group of automorphisms of the monster model as defined in [2].

Bibliography

[1] Mike Haskel and Anand Pillay, *On maximal stable quotients of definable groups in NIP theories*, Journal of Symbolic Logic, 83.1, pp. 117-122, 2018.

[2] Ehud Hrushovski, Krzysztof Krupiński, and Anand Pillay, *On first order amenability*, 2021, arXiv: 2004.08306 [math.LO].

The externally definable Ramsey property

Rob Sullivan

University of Münster

In this talk, I will introduce a weakened version of the Ramsey property: the "externally definable Ramsey property", where the colourings considered are restricted to those that are externally definable. We will explore several examples (and non-examples) of ultrahomogeneous structures with this property, and we will also discuss how to characterise ultrahomogeneous structures with the externally definable Ramsey property in terms of their topological dynamics.

This is joint work with Nadav Meir.

Geometry of Differentially Large Fields

Mikolaj Widawski

University of Manchester

In a paper from 2020, Johnson, Tran, Walsberg and Ye have introduced the étale-open topology on arbitrary fields, which is non-discrete if and only if the field is large and in the case of separably closed, real closed and non-separably closed valued fields recovers the natural topology. Methods arising from this topology were used by the authors to solve the stable fields conjecture for large fields, i.e., they prove that a large stable field is separably closed. This gives empirical evidence that methods involving the étale-open topology will be fruitful in the study of fields. In this talk we will introduce the concept of largeness and its analogue in the world of differential fields. The main result we will present will be a characterisation of differentially large Fields using the étale-open topology. This is closely related to a theorem of Guzy and Point (2009) on model completions of differential topological fields.

Generalised ultraproduct for positive logic

Kentaro Yamamoto

Czech Academy of Sciences

A generalisation of ultraproducts called prime products will be introduced. While ultraproducts preserve sentences of first-order logic (Łoś), prime products preserve sentences of positive logic in the sense of Poizat. The main result is an analogue of the Keisler-Shelah Theorem: two structures in the same language have the same positive theory if and only if some prime product of ultrapowers of one is isomorphic to some prime product of ultrapowers of the other.

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