

CATEGORIES LIKE HILBERT SPACES – HANDOUT 3

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Definition 1. A *central idempotent* in a monoidal category is a morphism $u: U \rightarrow I$ together with a natural transformation $\sigma_A: U \otimes A \rightarrow A \otimes U$ satisfying:

$$\begin{aligned} \rho \circ (\text{id}_U \otimes u) &= \lambda \circ (u \otimes \text{id}_U) & : U \otimes U &\rightarrow U \\ \text{id}_U \otimes u &\text{ is invertible} & U \otimes U &\rightarrow U \otimes I \\ \sigma_{A \otimes B} &= \alpha \circ (\text{id}_A \otimes \sigma_B) \circ \alpha^{-1} \circ (\sigma_A \otimes \text{id}_B) \circ \alpha & : U \otimes (A \otimes B) &\rightarrow (A \otimes B) \otimes U \\ \rho \circ (\text{id}_A \otimes u) \circ \sigma_A &= \lambda \circ (u \otimes \text{id}_A) & : U \otimes A &\rightarrow A \end{aligned}$$

Definition 2. An object S in a cartesian category is *subterminal* if the unique map $S \rightarrow 1$ is monic.