STOCHASTIC CONTROL, AMBIGUITY AND GAMES

LEEDS, 4TH–5TH SEPTEMBER 2017

List of Abstracts

1. Invited talks (alphabetical order)

Speaker: Ania Aksamit, University of Oxford
Title: Robust pricing–hedging duality for American options in discrete time financial markets
Abstract: We investigate pricing–hedging duality for American options in discrete time financial models where some assets are traded dynamically and others, e.g. a family of European options, only statically. In the first part we consider an abstract setting, which includes the classical case of a fixed probability measure as well as the robust framework with a non-dominated family of probability measures. Our first insight is that by considering a (universal) enlargement of the space, we can see American options as European options and recover the pricing–hedging duality, which may fail in the original formulation. This may be seen as a weak formulation of the original problem. Our second insight is that lack of duality is caused by the lack of dynamic consistency and hence a different enlargement with dynamic consistency is sufficient to recover duality: it is enough to consider (fictitious) extensions of the market in which all the assets are traded dynamically. In the second part of the paper we study two important examples of robust framework: the setup of Bouchard & Nutz [1] and the martingale optimal transport setup, see Beiglböck et al. [2], and show that our general results apply in both cases and allow us to obtain pricing–hedging duality for American options. (Based on joint work with S. Deng, J. Obłój and X. Tan.)

References
[1] Bouchard, B. and Nutz, M. (2015) *Arbitrage and Duality in Nondominated Discrete-Time Models*, Ann. Appl. Probab. **25**(2), pp. 823–859.
[2] Beiglböck, M., Cox, A.M.G. and Huesmann, M. (2016) *Optimal transport and Skorokhod embedding*, Invent. Math. doi:10.1007/s00222-016-0692-2.

Speaker: Natalie Caruana, University of Malta
Title: Nash Equilibrium in Nonzero-Sum Optimal Stopping Games for Brownian Motion
Abstract: We present solutions to nonzero-sum optimal stopping games for Brownian motion in $[0,1]$ absorbed at either 0 or 1. The approach used is based on the double partial superharmonic characterisation of the value functions derived in [1]. In this setting this characterisation of the value functions has a transparent geometrical interpretation of 'pulling two ropes' above 'two obstacles' which must however be constrained to pass through certain regions. This is an extension of the analogous result derived by Peskir in [2] and [3] (semiharmonic characterisation) for the value function in zero-sum optimal stopping games. To obtain the value functions and the corresponding optimal hitting times we transform the game into a free-boundary problem. The latter is then solved by making use of the double smooth fit

principle which was also observed in [1]. Under further assumptions on the payoff functions we prove that the solution to the free-boundary problem is unique, however when these assumptions are relaxed we provide examples to show that in general there can be more than one solution. We next provide a counterexample to show that if the initial assumptions on the payoff functions are relaxed then one may not be able to find a Nash equilibrium of the threshold type via the double partial superharmonic characterisation of the value functions. We conclude by explaining how the results obtained for absorbed Brownian motion extend to one-dimensional absorbed regular diffusions.

References

[1] Attard, N. (2015) *Nonzero-sum games of optimal stopping for Markov processes.* Research Report No. 1, Probab. Statist. Group Manchester. To appear in Appl. Math. Optim.
[2] Peskir, G. (2008) *Optimal stopping games and Nash equilibrium.* Theory Probab. Appl. 53 (558-571).
[3] Peskir, G. (2012) *A duality principle for the Legendre transform.* J. Convex Anal. 19 (609-630).

Speaker: Alexander M.G. Cox, University of Bath
Title: Robust Hedging of Options on a Leveraged Exchange Traded Fund
Abstract: A leveraged exchange traded fund (LETF) is an exchange traded fund that uses financial derivatives to amplify the price changes of a basket of goods. In this paper, we consider the robust hedging of European options on a LETF, finding model-free bounds on the price of these options.

To obtain an upper bound, we establish a new optimal solution to the Skorokhod embedding problem using methods introduced in Beiglboeck, Cox and Huesmann. This stopping time can be represented as the hitting time of some region for Brownian motion, but unlike the solutions of e.g. Root, this region is not unique. As a result, we show how to characterise the choice of the embedding region that gives the required optimality property. An important part of determining the optimal region is identifying the correct form of the dual solution, which has a financial interpretation as a model-independent super-hedging strategy. (Joint with Sam Kinsley).

Speaker: Erik Ekström, University of Uppsala
Title: Dynkin games with asymmetric information
Abstract: We study the effects of asymmetry of information for a two-person zero-sum optimal stopping game with linear payoffs. The drift of the underlying diffusion process is unknown to one player, but known to the other one. However, employing a Bayesian approach, the uninformed player has an initial prior distribution for the drift, and may learn about the drift from observations of the process and from the actions (or lack of actions) of

the informed player. We show that there exists a Nash equilibrium for this game, where the uninformed player uses a stopping time and the informed player uses a randomized stopping time in order to hide their informational advantage. This is joint work with Kristoffer Glover.

Speaker: Said Hamadene, University of Le Mans
Title: On the existence of a value of a Zero-sum switching games with general switching costs
Abstract: In this talk we discuss the problem of existence of a value for the zero-sum switching game in the markovian framework. The switching costs are not constant and can depend on time and the diffusion process as well. We show that the game has value. The techniques combine the notion of backward stochastic differential equation and viscosity solutions of systems of PDEs. This is a joint work with Marcus Olofsson, University of Uppsala, Sweden.

References

[1] Djehiche, B., Hamadene, S., Morlais, M.-A. (2015) *Viscosity solutions of systems of variational inequalities with interconnected bilateral obstacles*, Funkcialaj Ekvacioj, **58**, pp. 135–175.
[2] Djehiche, B., Hamadene, S., Morlais, M.-A., Zhao X. (2017) *On the equality of solutions of max-min and min-max systems of variational inequalities with interconnected bilateral obstacles*, Journal of Mathematical Analysis and Applications, **452**, pp. 148–175.
[3] Tang S., Hou, S.-H. (2007) *Switching games of stochastic differential systems*, SIAM Journal on Control and Optimization, **46**, pp. 900–929.

Speaker: David Hobson, University of Warwick
Title: TBA
Abstract: TBA

Speaker: Goran Peskir, University of Manchester
Title: Nonlinear Optimal Stopping and Nonlinear Optimal Control
Abstract: I will discuss problems of (a) optimal stopping and (b) optimal stochastic control in which the performance criteria are expressed by means of nonlinear functionals of the expected values. This leads to spatial/temporal inconsistencies which require novel concepts of solution. The three solution concepts known presently can be classified in terms of commitment to (i) the past (time-inconsistent), (ii) the future (Strotz subgame-perfect Nash equilibrium), (iii) the present (dynamic optimality). Discussion will be illustrated through optimal mean-variance portfolio selection problems. This will include a description of the first known time-consistent solutions to the constrained versions of these problems in continuous time (dating back to Markowitz for a single period model in the 1950s).

Speaker: Catherine Rainer, University of Brest
Title: A two player zerosum game where only one player observes a Brownian motion
Abstract: We study a two-player zero-sum game in continuous time, where the payoff -a running cost- depends on a Brownian motion. This Brownian motion is observed in real time by one of the players. The other one observes only the actions of his/her opponent. We prove that the game has a value and characterize it as the largest convex subsolution of a Hamilton-Jacobi equation on the space of probability measures. (joint work with Fabien Gensbittel, TSE Toulouse)

Speaker: Frank Riedel, University of Bielefeld
Title: Ambiguous Acts in Games
Abstract: We review a new approach to modelling ambiguous acts in games. By rigorously rooting the approach in decision theory, we provide a foundation for applications of Knightian uncertainty in mechanism design, principal agent and moral hazard models. We discuss critical assessments and provide alternative interpretations of the new equilibria in terms of equilibrium in beliefs, and as a boundedly rational equilibrium in the sense of a population equilibrium. We also discuss the purification of equilibria in the spirit of Harsanyi. Finally, we go on to discuss dynamic games under ambiguity.

Speaker: H. Mete Soner, ETH Zurich
Title: Singular Control in Economics
Abstract: Singular optimal control has been widely used in mathematical economics as a modelling tool. The classical paper of Constantinides [1] extends the utility maximization problem of Merton to markets in which there are proportional trading costs [4]. In the paper of [2] it provides a mechanism to investigate the firm value in corporate finance. Also these stochastic optimal control problems are naturally connected to free boundary problems. Indeed, the dynamic programing approach to these problems result in a nonlinear partial differential equation of the form

$$\max\left\{\beta v(x) + H(x, Dv(x), D^2 v(x))\ ;\ F(x, D(v(x)))\ \right\} = 0,$$

where $\beta > 0$, H and F are given nonlinear functions and H is non-increasing in the Hessian variable (i.e., it is elliptic). Hence, the solution v satisfies a gradient constraint and a free boundary determines the region in which this constraint is saturated.

In this talk, I will outline the optimal control problem and describe a recent application [3].

References

[1] Constantinides, G. M. (1986) *Capital Market Equilibrium with Transaction Costs*, J. Political Economy, **94**(4), pp. 842-862.

[2] Décamps, J-P, Mariotti, T., Rochet, J-C., and Villeneuve, S. (2011) *Free Cash-Flow, Issuance Costs and Stock Price Volatility*, Journal of Finance, **66**(5), pp. 1501-1544.

[3] Reppen, M., Rochet, J-C., and Soner, H.M. (2017) *Optimal dividends with random profitability*, preprint.

[4] Shreve, S. E. and Soner, H. M. (1994) *Optimal investment and consumption with transaction costs*, **4**(3), pp. 609–692.

Speaker: Yavor Stoev, University of Michigan
Title: Martingale optimal transport with stopping
Abstract: We solve the martingale optimal transport problem for cost functionals represented by optimal stopping problems. The measure-valued martingale approach developed in ArXiv: 1507.02651 allows us to obtain an equivalent infinite-dimensional controller-stopper problem. We use the stochastic Perron's method and characterize the finite dimensional approximation as a viscosity solution to the corresponding HJB equation. It turns out that this solution is the concave envelope of the cost function with respect to the atoms of the terminal law. We demonstrate the results by finding explicit solutions for a class of cost functions. Joint work with Erhan Bayraktar (UM) and Alex Cox (Bath)

2. Contributed talks (alphabetical order)

Speaker: Nacira Agram, University of Oslo
Title: Model Uncertainty Stochastic Mean-Field Control
Abstract: We consider the problem of optimal control of a mean-field stochastic differential equation (SDE) under model uncertainty. The model uncertainty is represented by ambiguity about the law $\mathcal{L}(X(t))$ of the state $X(t)$ at time t. For example, it could be the law $\mathcal{L}_{\mathbb{P}}(X(t))$ of $X(t)$ with respect to the given, underlying probability measure $\mathbb{P}$. This is the classical case when there is no model uncertainty. But it could also be the law $\mathcal{L}_{\mathbb{Q}}(X(t))$ with respect to some other probability measure $\mathbb{Q}$ or, more generally, any random measure $\mu(t)$ on $\mathbb{R}$ with total mass 1.

We represent this model uncertainty control problem as a *stochastic differential game* of a mean-field related type SDE with two players. The control of one of the players, representing the uncertainty of the law of the state, is a measure-valued stochastic process $\mu(t)$ and the control of the other player is a classical real-valued stochastic process $u(t)$. This optimal control problem with respect to random probability processes $\mu(t)$ in a non-Markovian

setting is a new type of stochastic control problems that has not been studied before. By introducing operator-valued backward stochastic differential equations (BSDE), we obtain a sufficient and a necessary maximum principle for Nash equilibria for such games in the general nonzero-sum case, and for saddle points in zero-sum games.

As an application we find an explicit solution of the problem of optimal consumption under model uncertainty of a cash flow described by a mean-field related type SDE.
(Joint with Bernt Øksendal)

Speaker: Alessandro Balata, University of Leeds
Title: Regress Later Monte Carlo for Optimal Control of Markov Chains
Abstract: A common approach in numerical solution of optimal control of discrete-time Markov processes is to discretise the process as a finite state space controlled Markov chain. This method suffers from the course of dimensionality and requires complex methods for design of efficient discretisation. On the contrary, Monte Carlo methods are not affected by the dimensionality of the state space. A successful adaptation of this approach to pricing of American option was performed in [4] and sparked a succession of papers. However, there it was crucial that the control (stopping) does not affect the dynamics of the underlying process. Adaptation of this approach to the problem of optimal control of discrete-time Markov processes was only achieved in [3], where the control was treated as an additional variable in the state space (control randomisation), but the convergence of the proposed scheme was not proved.

In this talk, I will present our approach in which we do not use control randomization. The key to our solution is the regress later approach developed to improve accuracy of American option pricing [2]. It consists of projecting the value function $V(t+1,x)$ over the linear space generated by basis functions $\{\phi_k(t+1,x)\}_{k=1}^{K}$ and then computing, possibly analytically,

$$\mathbb{E}[V(t+1,X_{t+1})|X_t=x,u_t=u]\approx\sum_{k=1}^{K}\alpha_k^{t+1}\mathbb{E}[\phi_k(t+1,X_{t+1})|X_t=x,u_t=u],$$

where u_t is the control at time t. The regress later approach allows us to: avoid randomisation of the control improving speed and accuracy; place training points freely leading to faster convergence; use performance iteration without resimulation saving computational time. It also allows us to prove that our numerical scheme converges, the task that seems very difficult for the control randomisation method [3].

In the talk I will introduce our approach, explain its links to existing solutions, sketch the convergence result and provide numerical evaluation. This talk extends our results in [1]. This is a joint work with Jan Palczewski.

References

[1] Balata, A., Palczewski, J. (2017) Regress-later Monte Carlo for optimal inventory control with applications in energy. arXiv:1703.06461, 2017.
[2] Glasserman, P., Yu, B. (2002) Simulation for American options: regression now or regression later? *In H. Niederreiter, editor, Monte Carlo and Quasi-Monte Carlo Methods*, pp. 213–226.
[3] Kharroubi, I., Langrené, N., Pham, H. (2014) A numerical algorithm for fully nonlinear HJB equations: an approach by control randomization. *Monte Carlo Methods and Applications* **20**(2), pp. 145–165
[4] Longstaff, F. A., Schwartz, E. S. (2001) Valuing American options by simulation: A simple least-squares approach. *Review of Financial Studies* **14**(1), pp. 113–147

Speaker: Daniel Bartl, University of Konstanz
Title: Pointwise superhedging duality in continuous time
Abstract: In this talk we focus on the pathwise (model-free) superhedging duality in continuous time. Under the assumption that the claim is an upper semicontinuity functional on the path space Omega (a subset of the continuous functions), we prove that the minimal superhedging price of the claim equals the supremum over its expectation under all martingale measures on Omega.

Joint work with Michael Kupper, David Prömel, and Ludovic Tangpi.

Speaker: Leonid Bogachev, University of Leeds
Title: Liouville-type theorems for the archetypal equation with rescaling
Abstract: In this talk, we consider a linear functional-integral equation

$$y(x) = \int\!\!\int_{\mathbb{R}^2} y(a(x-b))\,\mu(da,db), \qquad x \in \mathbb{R},$$

where μ is a probability measure on $\mathbb{R}^2$; equivalently, $y(x) = \mathbb{E}\{y(\alpha(x-\beta))\}$, with random (α, β) and $\mathbb{E}$ denoting expectation. This is a rich source of various functional and functional-differential equations with rescaling (hence the name *archetypal*), including the integrated Cauchy equation $y(x) = \mathbb{E}\{y(x-\beta)\}$ (i.e., $\alpha \equiv 1$) and the functional-differential ('pantograph') equation $y'(x) + y(x) = \mathbb{E}\{y(\alpha(x-\gamma))\}$.

Interpreting solutions $y(x)$ as harmonic functions of the associated Markov chain (X_n), we discuss Liouville-type theorems asserting that any bounded continuous solution is constant. The results crucially depend on the criticality parameter $K := \mathbb{E}\{\ln|\alpha|\}$; e.g., if $K < 0$ then a Liouville theorem is always true, but the case $K \geq 0$ is more interesting (and difficult). The proofs utilize the iterated equation $y(x) = \mathbb{E}\{y(X_\tau)|X_0 = x\}$ (with a suitable stopping time τ) due to Doob's optional stopping theorem applied to the martingale $y(X_n)$.

Random processes associated with the archetypal equation, in particular diffusions with multiplicative jumps, have a potential to find meaningful applications in financial mathematics, including appropriate optimal stopping problems. The speaker would be keen to receive feedback from the audience in this direction.

This is joint work with Gregory Derfel (Beer Sheva) and Stanislav Molchanov (UNC-Charlotte).

Speaker: Katia Colaneri, University of Perugia
Title: Portfolio optimization for a large investor controlling market sentiment under partial information
Abstract: The influence of large investors, such as hedge funds, mutual funds, and insurance companies, on prices of risky assets can be studied from very different viewpoints such as direct price impact from order execution (selling or buying) to feedback effects from trading to hedge portfolios of derivatives. There is also an influence of large investors on the overall market sentiment that arises from their perceived informational superiority. However, it is difficult, even for a large investor, to observe the exact state of the overall market and its effect on the price of the underlying risky asset and hence to act accordingly. Not knowing the exact state of the environment naturally necessitates a partial information setting, in which the large investor only observes the underlying risky asset but not the state of the environment. In this work we consider an investor faced with the utility maximization problem in which the stock price process has pure-jump dynamics affected by an unobservable continuous-time finite-state Markov chain, the generator of which depends on the choices of the investor. Using filtering results, we reduce the problem with partial information to one with complete information and solve it for logarithmic and power utility preferences. In particular, we apply control theory for piecewise deterministic Markov processes (PDMP) to our problem: we derive the optimality equation for the value function and characterize the value function as the unique viscosity solution of the naive Hamilton-Jacobi-Bellman equation. Finally, we provide numerical experiments to discuss how investor's ability to control the state process affects the optimal portfolio strategies as well as the optimal wealth under both partial and complete information.

References
[1] Altay, S., Colaneri, K., Eksi, Z. (2017). *Portfolio optimization for a large investor controlling market sentiment under partial information.* Available on SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2986612.

Speaker: Randall Martyr, Queen Mary University of London
Title: Nonzero-sum optimal stopping games and generalised Nash equilibrium problems

Abstract: In this talk, we will establish a duality between Nash equilibria for nonzero-sum games of optimal stopping (Dynkin games) and generalised Nash equilibrium problems (GNEP). In the Dynkin game this reveals novel equilibria of threshold type and of more complex types, and also connects convergence in an approximation scheme for equilibria to local stability in the GNEP.

This is joint work with John Moriarty (Queen Mary University of London), and is supported by EPSRC grants EP/K00557X/2 and EP/N013492/1.

Speaker: Eyal Neuman, Imperial College
Title: Incorporating Signals into Optimal Trading
Abstract: Optimal trading is a recent field of research which was initiated by Almgren, Chriss, Bertsimas and Lo in the late 90's. Its main application is slicing large trading orders, in the interest of minimizing trading costs and potential perturbations of price dynamics due to liquidity shocks. The initial optimization frameworks were based on mean-variance minimization for the trading costs. In the past 15 years, finer modelling of price dynamics, more realistic control variables and different cost functionals were developed. The inclusion of signals (i.e. short term predictors of price dynamics) in optimal trading is a recent development and it is also the subject of this work.

We incorporate a Markovian signal in the optimal trading framework which was initially proposed by Gatheral, Schied, and Slynko [2] and provide results on the existence and uniqueness of an optimal trading strategy. Moreover, we derive an explicit singular optimal strategy for the special case of an Ornstein-Uhlenbeck signal and an exponentially decaying transient market impact. The combination of a mean-reverting signal along with a market impact decay is of special interest, since they affect the short term price variations in opposite directions.

Later, we show that in the asymptotic limit were the transient market impact becomes instantaneous, the optimal strategy becomes continuous. This result is compatible with the optimal trading framework which was proposed by Cartea and Jaimungal [1].

This is a joint work with Charles-Albert Lehalle.

References
[1] Á. Cartea and S. Jaimungal. (2013) *Modelling asset prices for algorithmic and high-frequency trading.* Applied Mathematical Finance **20**(6), pp. 512–547.
[2] J. Gatheral, A. Schied, and A. Slynko. (2012) *Transient linear price impact and Fredholm integral equations.* Math. Finance **22**, pp. 445–474.

3. Posters (alphabetical order)

Speaker: Nahuel Foresta, Politecnico di Milano
Title: System of reflected BSDE driven by marked point processes and optimal switching
Abstract: We solve a switching problem in a marked point process (MPP) framework, where to each mode corresponds a different compensator that defines the dynamic. Such compensators form a dominated family.
Let $p(dsde)$ be a non-explosive MPP, and let $\phi_s(de)dA_s$ be its compensator under some fixed probability $\mathbb{P}$. For $i = 1, \dots, m$ consider ρ^i predictable random fields on $[0, T] \times E$. For each fixed switching strategy $\mathbf{a} = (\theta_n, \alpha_n)_{n \geq 1}$, consider the "switched compensator"

$$\rho_s^{\mathbf{a}} = \sum_{k \geq 1} \mathbb{1}_{(\theta_k, \theta_{k-1}]}(s) \rho_s^{\alpha_k}(e).$$

Through a Girsanov-like transform, $\rho^{\mathbf{a}}$ induces a new probability $\mathbb{P}^{\mathbf{a}} \ll \mathbb{P}$ under which the point process has compensator $\rho_s^{\mathbf{a}}(e)\phi_s(de)dA_s$. The optimal switching problem is thus given in a "weak form":

$$v(t, i) = \operatorname*{ess\,sup}_{\mathbf{a} \in \mathcal{A}_t^i} \mathbb{E}^{\mathbf{a}} \left[\xi^{a_T} + \int_t^T f_s^{a_s} dA_s - \sum_{k \geq 1} C_{\theta_k}(\alpha_{k-1}, \alpha_k) \,\middle|\, \mathcal{F}_t \right],$$

where ξ^i and f^i are the rewards, $C(i, j)$ are the costs of switching and a_t is the process indicating the current mode.
In order to solve this we introduce a system of reflected BSDE, driven by MPP, with interconnected obstacles. The value function is characterized as solution to this system. An optimal strategy is identified as the sequence of times when the "active" solution to the system hits the barrier.

Speaker: Min Gao, University of Manchester
Title: The British Binary Options
Abstract: We present a new class of binary options where the holder enjoys the early exercise feature of American binary options with his payoff is the 'best prediction' of the European binary payoff under the hypothesis that the true drift equals a contract drift μ_c. Based on the observed price movements, the option holder finds that the true drift of the stock price is unfavourable then he can substitute it with the contract drift. The key to the British binary option is the protection feature and to minimise the losses. A closed form expression for the arbitrage-free price is derived in terms of the rational exercise boundary and the rational exercise boundary itself can be characterised as the unique solution to a nonlinear integral equation. We also analyse the financial meaning of the British binary options using the results above.

Speaker: Jun Maeda, University of Warwick
Title: A Market Driver Volatility Model Via Policy Improvement Algorithm
Abstract: In the over-the-counter market in derivatives, we sometimes see large numbers of traders taking the same position and risk. When there is this kind of concentration in the market, the position impacts the pricings of all other derivatives and changes the behaviour of the underlying volatility in a nonlinear way.

We model this effect using Heston's stochastic volatility model modified to take into account the impact. The impact can be incorporated into the model using a special product called a market driver, potentially with a large face value, affecting the underlying volatility itself. We derive a revised version of Heston's partial differential equation which is to be satisfied by arbitrary derivatives products in the market. This enables us to obtain valuations that reflect the actual market and helps traders identify the risks and hold appropriate assets to correctly hedge against the impact of the market driver. (Joint work with S. Jacka, University of Warwick)

Speaker: Benjamin A. Robinson, University of Bath
Title: Stochastic Optimal Control Problems Related to Martingale Optimal Transport
Abstract: Martingale Optimal Transport (MOT) is a variant on the classical Monge-Kantorovich optimal transport problem, with the additional constraint that the initial measure evolves onto the target measure according to a martingale. In one dimension, it is known that a MOT problem can be reformulated as an optimal stopping problem for Brownian motion. We are interested in the higher dimensional case, where a natural analogue is a stochastic control problem, in which we optimise over all martingales of unit quadratic variation.

We investigate some control problems arising in this way, characterising the value functions as viscosity solutions to Hamilton-Jacobi-Bellman (HJB) equations, and noting a connection to the Monge-Ampère equation, which arises in the study of optimal transport. We also present some concrete examples, where the value function can be computed explicitly.

This research is supported by a scholarship from the EPSRC Centre for Doctoral Training in Statistical Applied Mathematics at Bath (SAMBa), under the project EP/L015684/1.

Speaker: Patrick Schuhmann, University of Bielefeld
Title: On a Dividend Problem with Capital Injection over a Finite Horizon
Abstract: In the seminal papers [1] and [2] Nicole El Karoui and Ioannis Karatzas established the connection between the so-called reflected follower stochastic control problem of a Brownian motion and an optimal stopping problem with absorption at zero. In this work we prove a similar link in the case in which the controlled process is a Brownian motion with drift, and

its reflection at zero is costly. These results pave the way for analysing an optimal dividend problem with capital injection over a finite time-horizon.

Joint work with G. Ferrari, University of Bielefeld.

References

[1] El Karoui, N., Karatzas, I. (1988). Probabilistic aspects of finite-fuel, reflected follower problems. Acta Applicandae Mathematicae, pp. 223–258.
[2] El Karoui, N., Karatzas, I. (1989). Integration of the optimal risk in a stopping problem with absorption. Séminaire de Probabilités XXIII, pp. 405–420.